(1) *Finite dimensional modules of a Hopf algebra*

Let $H$ be a Hopf algebra with invertible antipode $S$. Show that any finite dimensional $H$-module has a left and right dual.

(2) *Tangle categories and link invariants*

(a) Let $F_C : \text{Tang}^q_{fr}(V_1) \to \mathbb{C} = U_q\mathfrak{sl}_2\text{-mod}$ be the functor defined at the end of Lecture 20. Using the isomorphism

$$f : V_1 \to V_1^*$$

$$v_1 \mapsto \hat{v}_1$$

$$v_{-1} \mapsto -q^{-1}\hat{v}_1,$$

find $F_C$ on the cup and cap, i.e. find the maps $F_C(\cup) : \mathbb{C}(q) \to V_1 \otimes V_1$ and $F_C(\cap) : V_1 \otimes V_1 \to \mathbb{C}(q)$. Compute the value of $F_C(S^1) : \mathbb{C}(q) \to \mathbb{C}(q)$. Do you see any resemblance between this and the defining relations of the Kauffman bracket?

**Definition.** A *simple* module $S$ is a non-zero module whose only submodules are $\{0\}$ and $S$.

(3) *Representations of Sweedler’s Hopf algebra*

(a) Let $H$ denote Sweedler’s Hopf algebra defined in Exercise 2 on Sheet 10. Find all (up to isomorphism) simple $H$-modules.

**Hint: If a module is 1-dimensional, it is simple.**

(b) Prove that the tensor product of two simple $H$-modules is again simple.

(4) *Outlook/Reading exercise: Open-closed 2-TFTs*

There is a notion of *open-closed 2-TFTs* where one considers objects to be 1-dimensional (compact, oriented) manifolds possibly with boundary, i.e. finite disjoint unions of circles and intervals. In an analogous way to how 2-TFTs are given by commutative Frobenius algebras, one can show that open-closed 2-TFTs correspond to so-called *knowledgable Frobenius algebras*. Work through Example 4.1.14 in [https://www.math.uni-hamburg.de/home/schweigert/ws12/hskript.pdf](https://www.math.uni-hamburg.de/home/schweigert/ws12/hskript.pdf) to understand this statement better.

\footnote{Initially the “hats” were wrong in the last expression, but it is now correct.}