Summer semester 2021

Claudia Scheimbauer
Eilind Karlsson

Bordisms and TFTs - Exercise 9

(1) *Example of a R-matrix*

Let V be a 2 dimensional vector space. Let $R \in \mathrm{End}(V \otimes V)$ be given by

$$R(q,\lambda) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \lambda & 1-q\lambda & 0 \\ 0 & 1-q^{-1}\lambda & \lambda & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

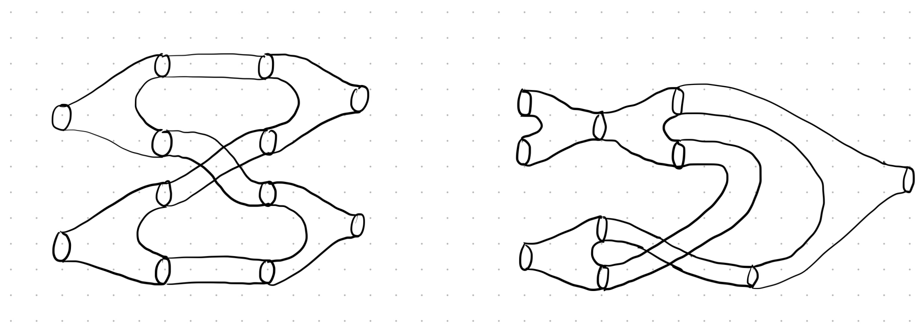
where q is a non-zero scalar and λ is an arbitrary scalar. Show that $R(q,\lambda)$ satisfies the Yang-Baxter equation if

$$\lambda + \lambda^3 = (q+q^{-1})\lambda^2.$$

Remark: This R-matrix appears in the *six vertex model*, which is one example of an *integrable lattice model*. For more details see e.g. Section 4.3 of `https://www.math.uni-hamburg.de/home/schweigert/ws12/hskript.pdf`.

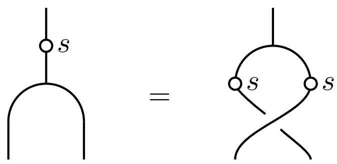
(2) *Bringing 2 dimensional cobordisms to normal form*

(a) Use the relations of a (co)commutative Frobenius algebra step by step to bring the below two images to normal form:



(3) *The antipode is an anti-endomorphism.*

(a) Let $\mathcal{C}$ denote a braided strict monoidal category, and let $(H, \mu, \eta, \Delta, \varepsilon)$ be a Hopf algebra in $\mathcal{C}$ with antipode $s : H \to H$. Prove that s is an anti-endomorphism of H as an algebra, i.e. show that



Hint: See the bottom of the page.

(b) Is there a similar relation involving the antipode and comultiplication?

Hint for Exercise 3: Simplify the following composition in two different ways

$$\mu \circ (\mu \otimes \mathrm{id}) \circ (\mu \otimes \mathrm{id} \otimes s) \circ (s \otimes s \otimes \Delta) \circ (\beta \otimes \mu) \circ (\mathrm{id} \otimes \beta \otimes \mathrm{id}) \circ (\Delta \otimes \Delta) : H \otimes H \to H.$$