

# Combinatorics of Coxeter Groups

## Learning aid for the exam

The following is a non-exhaustive list of notions and topics that can be asked about in the exam. It may be helpful to keep connections between these topics as well as explicit examples in mind. Section numbers refer to our standard reference, *Combinatorics of Coxeter groups* by Andreas Björner and Francesco Brenti.

- Definition of Coxeter groups (1.1)
- The exchange property and its consequences (1.4 and 1.5)
- The explicit description of Coxeter groups of types  $A_n$ ,  $B_n$  and  $I_2(m)$  (1.2). Various computations (e.g. the length of an element) for types  $A_n$  (chapters 1 and 2) and  $B_n$  (8.1)
- The different characterizations of the Bruhat order (2.1 and 2.2)
- Lifting property of the Bruhat order (2.2)
- Parabolic subgroups, quotients, and their applications on the Bruhat order (2.4 – 2.6)
- The tableau criterion, i.e. deciding Bruhat order for type  $A_n$  (2.6)
- Properties of the longest element of a finite Coxeter group (2.3)
- The different characterizations of the weak order (3.1)
- Lattice properties of the weak order (3.2)
- The word property, and computation with reduced expressions in general Coxeter groups (3.3)
- Definition and properties of the geometric representation of a Coxeter group (4.1 and 4.2)
- The numbers game for general Coxeter groups, and its relation to the geometric representation (4.3)
- Definition and properties of the root system of a Coxeter group (4.4)
- The bijection between reflections and positive roots of a Coxeter group (4.4)
- The root poset, and the dual numbers game to compute it (4.6)
- Characterizations of the set of small roots (4.7)
- Construction of the canonical automaton (4.8)