

Combinatorics of Coxeter Groups

Exercises and reading assignment for June 22nd

Read sections 4.3 and 4.4. You may ignore the last example in section 4.3, as we did not discuss normal forms. While reading section 4.4, keep the example of the symmetric group S_3 in mind (Figure 4.2).

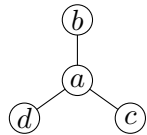
Exercise 1

Let (W, S) be a Coxeter group. Let $T \subset W$ be the set of reflections.

- (a) W is finite if and only if T is finite.
- (b) If W is finite and $w_0 \in W$ is its longest element, then $\ell(w_0)$ is the number of positive roots.

Exercise 2

Consider a Coxeter group (W, S) of type D_4 . We write $S = \{a, b, c, d\}$ with the following Coxeter graph:



- (a) What position of the numbers game, starting with the unit position $p = \mathbf{1}$, corresponds to the longest element $w_0 \in W$?
- (b) List the positive roots (Hint: Derive the number of positive roots from (a) and Exercise 1, then compute roots until you have got enough).