Technische Universität München
Zentrum Mathematik

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Winter term 2017/18
Exercise sheet 8

Linear algebraic groups (MA 5113)

Exercise 29. Consider the natural action of GL_n on $\mathbb{A}^n$. Compute the orbits, as well as the partial order defined in Exercise 11, of the following subgroups of GL_n and draw the orbits for $n = 2$.

(a) GL_n.
(b) B, the subgroup of upper triangular matrices.
(c) U, the subgroup of unipotent upper triangular matrices.
(d) T, the diagonal torus.

Exercise 30. Let U be a unipotent group over the (algebraically closed) field k. Show that

(a) if $\operatorname{char} k = 0$, then U is connected.
(b) if $\operatorname{char} k = p > 0$, then the number of irreducible components of U is a power of p.

Exercise 31. Let G be a linear algebraic group and let $G^{\mathrm{ab}} := G/G^{\mathrm{der}}$ and $\pi\colon G \to G^{\mathrm{ab}}$ the canonical projection. Show that G^{ab} is Abelian and satisfies the following universal property. For any homomorphism of linear algebraic groups $\varphi\colon G \to H$ with H Abelian there exists a unique morphism φ^{ab} such that the diagram

$$\begin{array}{ccc} G & \xrightarrow{\varphi} & H \\ {\scriptstyle \pi}\downarrow & \nearrow_{\varphi^{\mathrm{ab}}} & \\ G^{\mathrm{ab}} & & \end{array}$$

commutes.

Exercise 32 (Decomposition series)**.** Calculate the decomposition series of the following linear algebraic groups.

(a) B.
(b) GL_n.

You may use without proof that $SL_n(k)$ is generated by the elementary matrices $E_{i,j}(\lambda)$ for $1 \le i \ne j \le n$ and $\lambda \in k$, where $E_{i,j}(\lambda)$ denotes the matrix whose entries are 1 on the diagonal, λ at the (i, j)-coordinate and 0 otherwise.

Deadline: Friday, 15th December, 2017

If you have questions regarding the exercises, please send an email to hamacher@ma.tum.de.
The exercise classes are Fridays, 10-12 in room MI 02.08.020.
Further information about our lectures and exercises are available under

`http://www-m11.ma.tum.de/viehmann/viehmann-linear-algebraic-groups/`.