Technische Universität München
Zentrum Mathematik

Dr. Paul Hamacher

Winter term 2020/21
Exercise sheet 2

Abelian varieties (MA 5115)

Exercise 1 (The unit component)**.** Let G be an algebraic group and let $G^0 \subset G$ be the connected component containing e.

(a) Show that G^0 is an algebraic group and that it is normal in G.

(b) Show that all connected components of G are isomorphic as varieties

Exercise 2 (Kernel and image)**.** Let $\varphi\colon G_1 \to G_2$ be a homomorphism of algebraic groups.

(a) Show that $\ker\varphi := \{g \in G_1 \mid \varphi(g) = 0\} \subset G_1$ is a closed algebraic subgroup.

(b) Assume that G_1 is proper. Show $\operatorname{im}\varphi := \{\varphi(g) \mid g \in G_1\} \subset G_2$ is a closed algebraic subgroup. Can you show this without assuming that G_1 is proper ?

(c) Now assume that G_1 is an Abelian variety. Show that $(\ker\varphi)^0$ and $\operatorname{im}\varphi$ are also Abelian varieties.

(d) Assume that $\operatorname{char} k = p > 0$ and let $\mathbb{G}_m = (k\backslash\{0\}, \cdot)$ denote the multiplicative group. Calculate $\ker\varphi$ and $\operatorname{im}\varphi$ for $\varphi\colon \mathbb{G}_m \to \mathbb{G}_m, x \mapsto x^p$. Can you see a problem ? Can you solve it ?

Deadline: Monday, 16th November, 2020