Technische Universität München
Zentrum Mathematik

Dr. Paul Hamacher

Winter term 2020/21
Exercise sheet 6

Abelian varieties (MA 5115)

As usual, we work over an algebraically closed field k.

Exercise 1. Let A be an Abelian variety and $\mathscr{L} \in \operatorname{Pic} A$. We consider the map

$$\phi\colon A(k) \to \operatorname{Pic}(A), x \mapsto t_x^*\mathscr{L} \otimes \mathscr{L}^{-1}.$$

(a) Show that ϕ is a group morphism.
(b) Prove that

$$K(\mathscr{L}) \coloneqq \ker \phi \subset A(k)$$

is a closed in $A(k)$ and thus an Abelian variety.

Exercise 2. Let $A, \mathscr{L}, \phi$ be as in the previous exercise. We choose a divisor $D \in Z^1(A)$ such that $\mathscr{L} \cong \mathscr{O}(D)$.Prove that the following are equivalent:
(a) The subgroup $H \coloneqq \{x \in X \mid t_x^* D = D\}$ is finite
(b) $K(\mathscr{L})$ is finite
(c) $\mathscr{L}$ is ample
Here you may use that a divisor D is ample if and only if the global section of $\mathscr{O}(2D)$ define finite morphism $\varphi\colon A \to \mathbb{P}^N$.

Deadline: Monday, 14th December, 2020