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Seminar on *Special cycles on Shimura varieties*

We want to study questions on the geometry of Shimura varieties arising in the context of Kudla's program of relating intersections of cycles on Shimura varieties to Eisenstein series. In the first part of the seminar we will study two (of the few exceptional) examples where explicit descriptions of the basic locus of Shimura varieties can be given. In the first case, where the local group is isomorphic to GSp_4, the irreducible components are just projective lines, and also their intersections can be described explicitly. The second example considers the case of Spin groups, of particular interest due to its applications to moduli of K3 surfaces. The description in these cases uses the recent definition of integral models for Shimura varieties of Hodge type by Kisin.

In the last part of the seminar we want to understand how this can be applied to compute intersections of special cycles on the corresponding Shimura varieties. For this we return to the first, much easier example. (In the second example such a theory is not yet available.)

Talk 1. Shimura varieties, integral models and Newton stratification.
The first goal is to define Shimura varieties of PEL- and Hodge-type. Discuss then in detail the Shimura varieties attached Spinor groups from [KR97] §1, [HP15] §3.1. The second goal is to state the existence of canonical integral models of these Shimura varieties proven by Kisin [Ki13]. The final part of this talk should be devoted to introducing the Newton stratification (and in particular, the basic locus) on these Shimura varieties (cf. [Wo13]).

Talk 2. Explicit description of the supersingular locus.
The goal of this talk is to give a fairly explicit description of the supersingular locus (= basic Newton stratum) of a certain low-dimensional Shimura variety, following the presentations in [KR97] §4 and [Ka97] pp. 17-19.

Talk 3. Rapoport-Zink functor and affine Deligne-Lusztig varieties.
Define the Rapoport-Zink functor RZ_G^{fsm} attached to a local unramified Shimura-Hodge datum [HP15] §2.3 (in particular, 2.3.6). Study its field-valued points via (refined) affine Deligne-Lusztig varieties [HP15] §2.4.

Talk 4. Representability, uniformization and local Shimura data. (longer talk)
In this talk not all proofs should be given. One should discuss the following topics:

- Sketch the construction of the formal scheme RZ_G and the representability result [HP15] 3.2.9, which shows that the Rapoport-Zink functor RZ_G^{fsm} is represented by this formal scheme.
- Sketch the formal uniformization theorem of Kim [HP15] Section 3.3 (in particular, 3.3.2), which shows that the formal completion of the Shimura variety along the basic Newton stratum is uniformized by the Rapoport-Zink formal scheme RZ_G from the previous talk.
- Introduce the local Shimura datum attached to the group GSpin ([HP15] §4.2, the goal is 4.2.6 Proposition). Study the attached Rapoport-Zink space ([HP15] 4.3). In particular, show why the construction from talk 3 is applicable to this situation (cf. 4.3.2).

Talk 5. Description of the basic locus I. (longer talk) In this talk and the next one we want to understand the description of the superbasic locus in terms of classical Deligne-Lusztig varieties using a decomposition according to vertex lattices. Define vertex lattices, describe the vertex lattice attached to a point in the basic Rapoport-Zink space, and the corresponding decomposition into strata RZ_Λ. Describe these strata in terms of classical Deligne-Lusztig varieties. ([HP15], 5, 6.1-6.4)

Talk 6. Description of the basic locus II. Explain the relation to the Bruhat-Tits building ([HP15], 6.5). Apply the description of the Rapoport-Zink space to obtain a description of the supersingular locus of the corresponding Shimura variety as in [HP15], Theorem 7.3.3. Non-emptiness of the supersingular locus should not be proved. ([HP15], 7)

For the last three talks we return to the Shimura variety for the group GSp_4. Following Kudla and Rapoport we use the description of the basic locus to compute intersection numbers of so-called special cycles. The last two talks have to give an overview over a larger amount of material. In case of more participants in the seminar, they can be divided among several speakers. A general reference for background material is [ARGOS].

Talk 7. Special Cycles. Define special cycles (using the modular definition as in [KR97], 2) and their intersections. Prove their basic properties as in [KR97], 3.

Talk 8. The contribution of isolated points. The main goal of this talk is Theorem 7.2 which computes the contribution of the isolated points to the intersection number. Explain the theorem and its proof. Prop. 6.1 should be stated without proof. Parts of KR, 5 should be discussed (as much as time permits, possibly even in a separate talk) to understand the description of the set of isolated points.

Talk 9. The relation to Eisenstein series. The goal is to understand [KR97], Theorem 9.2. Explain the Eisenstein series and Whittaker functions involved in the analytic side of the comparison, and if time permits outline the key ideas of the proof ([KR97], 8-9, [K97]).

References

[ARGOS] U. Görtz, M. Rapoport (ed.): Argos-Seminar on intersection of modular correspondences, Asterisque 312 (2010).

[HP15] Howard B., Pappas G.: *Rapoport-Zink spaces for spinor groups*, preprint, 2015, http://arxiv.org/abs/1509.03914.

[K97] Kudla S.: *Central derivatives of Eisenstein series and height pairings*, Ann. Math. **146** (1997), 545–646.

[KR97] Kudla S., Rapoport, M.: *Cycles on Siegel threefolds and derivatives of Eisenstein series*, Ann. Sci. École Norm. Sup. **33** (2000), 695–756.

[Ki13] Kisin M.: *mod p points on Shimura varieties of abelian type*, preprint, 2013.

[Ka97] Kaiser C.: *Ein getwistetes Fundamentales Lemma für die* GSp_4, Dissertation, Bonn, 1997.

[Wo13] Wortmann D.: *The μ-ordinary locus for Shimura varieties of Hodge type*, preprint, 2013, http://arxiv.org/abs/1310.6444.