Teaching Portfolio

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Contents

1 Introduction

1.1 Teaching Philosophy

Aber der Lehrer muß den Mut haben, sich zu blamieren. Er muß sich nicht als der Unfehlbare zeigen, der alles weiß und nicht irrt, sondern als der Unermüdliche, der immer sucht und vielleicht manchmal findet. Warum Halbgott sein wollen? Warum nicht lieber Vollmensch?

A. Schönberg, "Harmonielehre", Wien 1911.

I believe that the goal of my teaching lies in passing on the foundations of my field, as well as to waken the interest in the quest for new knowledge. To achieve this, I believe it to be my task to provide students with the necessary background and skills to deal with classical problems, as well as to enable them to pursue future studies and research by themselves. Depending on the audience and the course, the emphasis may be shifted much more towards one of these goals only. Also, the various students' motivations may vary a lot and their goals may greatly differ from mine. I am aware that everyone learns at a different pace, I am aware that my way of approaching and teaching a subject is led by my own preferences and interests, and also, that I am not omniscient.

Therefore, combining knowledge with the search for it in a varying and diverse environment is important for my teaching, as well as my research. For me, this implies that as a lecturer and teacher, I have to play different rôles, sometimes even within the same lecture. But how can I implement this variety without frustrating or confusing my audience? Here, I believe that it is important to be clear about my goals and my expectations from the very beginning. I also try to make clear during my lectures where we are currently standing, where we came from, and where we are heading to - repetition is a key. Of course, service lectures will be much stricter and much more formal in style, where I consider myself to be much more of a leader, whereas in lectures at research level, I consider myself more of a scientist who is first among equals.

Finally, I believe that teaching always goes both ways - I learnt a lot about a subject from teaching it and sometimes, allegedly simple or naive questions from the students turned out to be deeper than I thought. As a result, I take feedbacks from my students seriously and even though I may not always agree with them, I will at least give their objections a thought.

So much for my rôle and my self-perception. But what about my expectations towards my students and their rôle? I believe that learning mathematics takes place in solitude and by solving exercises with pen and paper. Therefore, attending my lectures can only be a part of the process of learning and understanding a subject. I consider solving the weekly exercises, which come with my lectures, as well as participating in exercise classes and tutorials as equally important. The flows are thus as follows: In the lecture, I present and illustrate the material. In solitude, the student thinks and solves the exercises. In the tutorials, the student presents solutions, asks questions, and gets feedback. And then, the circle starts all over again...

1.2 Brief Biography – With A View Toward Teaching

2023	Sabbatical (6 months) at University of Oxford, UK *teaching:* supervising students
2022 - now	Professor (W3, full) at TU München *teaching:* lectures, seminars, supervising students,...
2017 - 2022	Professor (W3, associate) at TU München *teaching:* lectures, seminars, supervising students,...
2013 - 2017	Professor (W2, tenure track W3) at TU München *teaching:* lectures, seminars, supervising students,...
2012 - 2013	Postdoc (Privatdozent) at Universität Bonn *teaching:* a graduate lecture, a seminar, summer schools

Habilitation at Universität Düsseldorf, November 2011
Thesis: *Algebraic Surfaces and Geometry in Positive Characteristic*

2009 - 2011	Visiting Scholar (2 years) at Stanford University, USA *teaching:* a graduate lecture, a seminar, summer schools
2004 - 2009	Assistant (C1) at Universität Düsseldorf *teaching:* a service lecture, seminars, exercise classes

Ph.D. from Universität Bonn, November 2004
Advisor: Gerd Faltings
Thesis: *On Fundamental Groups of Galois Closures of Generic Projections*

2001 - 2004	Ph.D. student at Max-Planck Institut für Mathematik in Bonn

Diploma from Universität Göttingen, July 2001
Advisor: Fabrizio Catanese
Thesis: *Singular Abelian Covers of Algebraic Surfaces*

1998 - 1999	Erasmus Student (1 year) at University of Warwick, UK
1996 - 2001	Student at Universität Göttingen *teaching:* exercise classes

2 Main part

2.1 Teaching Experience

2.1.1 Lectures

Service lectures

Lineare Algebra für Informatik — TUM
1st semester, 4 hours/week, 8 ECTS points
in SS 2019 with 1073 students
in SS 2014 with 465 students

Mathematik für Physiker (Lineare Algebra) — TUM
1st semester, 4 hours/week, 8 ECTS points
in WS 2017/18 with 382 students

Undergraduate lectures (Bachelor level)

Algebra 2 — TUM
4th semester, 4 hours/week, 9 ECTS points
in WS 2021/22 with 43 students
(online, due to the Corona pandemic)
in SS 2015 with 33 students
in SS 2013 with 26 students

Algebra — TUM
4th semester, 5 hours/week, 9 ECTS points
in SS 2021 with 101 students
(online, due to the Corona pandemic)
3rd semester, 4 hours/week, 9 ECTS points
in WS 2016/17 with 103 students

Lineare Algebra 2 — TUM
2nd semester, 6 hours/week, 9 ECTS points
in SS 2020 with 219 students
(online, due to the Corona pandemic)

Lineare Algebra 1 — TUM
1st semester, 6 hours/week, 9 ECTS points
in WS 2019/20 with 219 students

Lineare Algebra 1 — Düsseldorf
1st semester, 4 hours/week, 10 ECTS points
in SS 2009 with approximately 50 students

Undergraduate lectures (Master level)

Elliptische Kurven — TUM
- 4^{th} or 6^{th} semester, 4 hours/week, 9 ECTS points
  - in SS 2016 with 16 students

Algebraische Geometrie — TUM
- 5^{th} semester, 4 hours/week, 9 ECTS points
  - in SS 2022 with 20 students
  - in WS 2015/16 with 21 students
  - in WS 2013/14 with 17 students

Algebraic Geometry 2 — Bonn
- 5^{th} semester, 4 hours/week, 9 ECTS points
  - in WS 2012/13 with approximately 20 students

Algebraic Geometry 2 — TUM
- 6^{th} semester, 4 hours/week, 6 ECTS points
  - in WS 2022/23 with 9 students

Graduate lectures

Algebraic Surfaces — TUM
- Ph.D. students, 4 hours/week, estimated 9 ECTS points
  - in SS 2018 with approximately 10 students

Algebraic Surfaces — Stanford
- Ph.D. students, 2 hours/week, estimated 4 ECTS points
  - in SS 2010 with approximately 15 students

See also Section 2.3 for lecture series given at summer schools.

2.1.2 Seminars

Seminars (Bachelor level)

Darstellungstheorie endlicher Gruppen TUM
3^{rd} semester, 2 hours/week, 5 ECTS points
in SS 2019 with 4 students
each giving 2 talks
Topologie TUM
3^{rd} semester, 2 hours/week, 5 ECTS points
in WS 2014/15 with 10 students
Lie-Algebren TUM
4^{th} semester, 2 hours/week, 5 ECTS points
in SS 2014 for 12 students

Seminars (Master level)

Toric Geometry TUM
5^{th} semester, 2 hours/week, 5 ECTS points
in SS 2022 for 10 - 14 students
Elliptic curves TUM
6^{th} semester, 2 hours/week, 5 ECTS points
in WS 2022/23 for 10 - 14 students
Klassische algebraische Geometrie TUM
5^{th} semester, 2 hours/week, 5 ECTS points
in WS 2013/14 for 10-14 students

Reading courses (Master or graduate level)

Algebraic and Complex Geometry TUM
$\geq 7^{th}$ semester, 2 hours/week
in WS 2016/17 with approximately 7 students
in SS 2016 with approximately 7 students
in WS 2015/16 with approximately 7 students
in SS 2015 with approximately 7 students
Algebraic Topology TUM
$\geq 7^{th}$ semester, 2 hours/week
in WS 2014/15 with approximately 7 students

These are reading courses that are organized by postdoctorial and Ph.D. students, who are usually reading an advanced text book on graduate level, partly under my supervision.

Seminars (Graduate level)

F-singularities WS 2021/22, 2 hours/week (online, due to the Corona pandemic)	LMU and TUM
The Merkurjev-Suslin Theorem SS 2021, 2 hours/week (online, due to the Corona pandemic)	LMU and TUM
Cubic Hypersurfaces SS 2020, 2 hours/week (online, due to the Corona pandemic)	Worldwide
Logarithmic Geometry SS 2019, weekend seminar in the Academy Center Raitenhaslach	LMU and TUM
Motivic Cohomology SS 2018, 2 hours/week	LMU and TUM
Rationality Questions in Algebraic Geometry WS 2017/18, 2 hours/week	LMU and TUM
Moduli Spaces of Enriques Surfaces SS 2016, 2 hours/week	LMU and TUM
Derived Categories of Coherent Sheaves SS 2015, 2 hours/week	LMU and TUM
Frobenius and Hodge Degeneration WS 2009/10, 2 hours/week	Stanford

These seminars are aimed at advanced master students, Ph.D. students, postdoctorial students, and professors. The topics cover material of current research. See also Section 2.3 for workshops.

Seminars (Graduate and postgraduate level)

Since SS 2013, I am also running an 'Oberseminar' (research seminar), where graduate students, postdoctorial students, and professors (external and from my own research group) give talks about their work. These seminars are 2 hours/week, sometimes co-organised with colleagues from TUM or LMU, sometimes on my own.

2.1.3 Supervised Theses

Bachelor theses

M. Burger	Nagata's Theorem for Group Schemes (thesis within the TopMath program)	2023	TUM
O. Zühlke	Die Mathieu-Gruppen	2023	TUM
J. Ott	Primärzerlegung	2023	TUM
D. Lang	Henselian Rings and Implicit Functions	2023	TUM
J. Schneidemann	Die globale Dimension von Ringen	2023	TUM
B. Kremser	Fano Schemata von Geraden	2022	TUM
D. Nobile	Gröbner Bases and Applications	2022	TUM
P. Boettger	Gleichungen kleinen Grades	2022	TUM
S. Spindler	The cde triangle and Brauer characters	2020	TUM
C. Esparza	Differentialoperatoren auf Schemata	2019	TUM
B. Kremser	Fano Schemata von Geraden	2019	TUM
M. Schneider	Die étale Fundamentalgruppe	2019	TUM
C. Stadlmayr	Deformations and Resolutions of Rational Double Points (thesis within the TopMath program)	2018	TUM
N. Gschwendtner	Locally Compact Fields	2016	TUM
A. Merkel	Profinite Groups and Infinite Galois Theory	2016	TUM
A. Pieper	Intersections of quadrics and Hessians of cubics	2016	TUM
J. Graw	de Rham Kohomologie und Maxwell Gleichungen (supervised jointly with Christian Pfleiderer (TUM), interdisciplinary with the department of physics)	2016	TUM
T. Dölling	Nash Blow-ups	2015	TUM
L. Huynh Huu	The 27 Lines on a Cubic Surface	2014	TUM
G. Martin	Automorphisms of Curves (thesis within the TopMath program)	2014	TUM
A. Trost	Quotient Singularities (thesis within the TopMath program)	2014	TUM
D. Kohn	Graßmann-Varietäten	2014	TUM
C. Faure-Brac	Lattice coding for Rayleigh fading channels (supervised jointly with Camilla Hollanti (Aalto))	2014	TUM
J. Bongartz	The Hesse pencil and its Cayleyan (supervised jointly with Daniel Huybrechts (Bonn))	2012	Bonn
C. Weiß	Die Hessesche Fläche einer kubischen Fläche (supervised jointly with Daniel Huybrechts (Bonn))	2012	Bonn

Practical projects (short theses between Bachelor and Master)

F. Neugebauer	The Weil conjecture for curves via Jacobian varieties (supervised jointly with Marius Kjærsgaard (Copenhagen))	2023	TUM
H. Chua	The abc conjecture	2022	TUM
A. Merkel	Auflösen von Gleichungen	2016	TUM
L . Panny	A primer on practical primality proving	2016	TUM

Interdisciplinary projects with computer science

C. Madlener	Formal verification of the RANKING algorithm for online bipartite matching (supervised jointly with Tobias Nipkow (TUM))	2022	TUM
R. Dordjonova	Formalising matching theory: characterisations of perfect matchings (supervised jointly with Tobias Nipkow (TUM))	2022	TUM

Master theses

X. Zhang	On q-Weil Numbers	2023	TUM
S. Spindler	Isogenies of elliptic curves over finite fields (supervised jointly with Jens Zumbrägel (Passau))	2023	TUM
B. Tu	Toric Fano Varieties and Fano Polytopes	2023	TUM
H. Chua	On Waring's Problem (supervised jointly with Nikita Semenov (LMU))	2022	TUM
K. Öztaş	Endomorphisms of Abelian Varieties	2021	TUM
C. Stadlmayr	Which rational double points occur on del Pezzo surfaces? (received the TopMath Award 2020)	2020	TUM
J. Graw	Braid Foliations in Lens Spaces (supervised jointly with Joan Licata (ANU))	2018	TUM
L. Panny	Efficient point counting and Monsky-Washnitzer cohomology	2017	TUM
C. Borger	On group schemes of order p^2	2016	TUM
G. Martin	On Extremal Enriques Surfaces (received the TopMath Award 2016)	2016	TUM
R. Riedl	Brauer-Severi Varieties	2015	TUM

Ph.D. theses

C. Stadlmayr	Geometry of rational double points and del Pezzo surfaces	2023	TUM
D. Boada	Moduli of Bielliptic Surfaces	2021	TUM
K. Behrens	Moduli of Supersingular Enriques Surfaces	2020	TUM
G. Martin	Automorphisms of Enriques Surfaces	2018	TUM

2.2 Teaching Concept

In the sequel, I will illustrate the implementation of my Teaching Philosophy from Section 1.1 by examples from the following two lectures

1.	Lineare Algebra für Informatik	SS 2014	TUM
2.	Elliptic Curves	SS 2016	TUM

The first is a service lecture that is compulsory for students of computer science in their first year. The second is a lecture on advanced mathematics, aimed particularly at students who consider writing a thesis over even research in this field. Thus, these two lectures are very different in level and audience, demanding different rôles from me and require different teaching approaches.

2.2.1 Generalities on Lectures in Mathematics

As common in mathematics, both of the above mentioned lectures consist of a lecturer (=me) teaching 4 hours per week with chalk at a blackboard, and I refer to Section 2.5 for my objections against slides and powerpoint. These are accompanied by weekly worksheets and 2 hours of example classes per week. There is an oral or a written exam at the end of term plus a second exam (resit) before the beginning of the next term.

Mathematics, and especially pure mathematics, deals with abstract structures, their classification, and computing in them. Due to the abstract nature of the subject, it is important for me to illustrate definitions with elementary examples and to be clear about whether a result is a technical lemma, an intermediate step, or a major classification result. I find it especially helpful to have small surveys - oral and less than 5 minutes - at the beginning and at the end of each lecture to repeat the achieved results and give short outlooks. These surveys form a sort of frame and give hopefully more grip, see also Section 2.5.

I believe that learning mathematics takes place in solitude and by solving exercises with pen and paper, see Section 1.1. Therefore, my lectures are accompanied by weekly worksheets and weekly example classes. In solving the worksheets and presenting their solutions in example classes, the students get a better feeling of what they really understand and whether they can make themselves understood. But the example classes also serve another purpose: whereas I can pose small comprehension questions in smaller lectures, this is almost impossible in larger lectures. Thus, especially in the latter, example classes are important to get feedback from the students. Of course, this feedback is usually via a tutor, but it also has the advantage that students are usually more open and frank to the tutor than they are to me.

Finally, in case of a written exam, I usually allow my students to bring a hand-written sheet of A4 paper to this exam, on which they are allowed to write whatever they want. In order to compose such a sheet of paper, the students have to contemplate which concepts and objects are important, and they have to condense the lecture.

2.2.2 Example 1: Lineare Algebra für Informatiker

title	Linear Algebra for Computer Science
audience	students of computer science in their first year
format	one semester with 4 hours of lecture and 2 hours of example classes per week
credits	8 ECTS points
language	German
assessment	weekly worksheets and a written exam (90 minutes) at the end of term

audience: This lecture is compulsory for students of computer science in their first year. In particular, there is usually a large audience (I had 465 students when teaching this course in SS 2014) and since this lecture contains abstract mathematical background, oftentimes with no immediate application to computer science, I did not except many students to be highly motivated or to be particularly interested in the subject. With this in mind, I wanted to have an emphasis on algorithms and recipes rather than on abstract concepts and proofs. Also, having a script was important to me.

learning outcomes and their assessment:

- natural numbers, integers, real and complex numbers

  The students can recall the definitions of natural numbers, the integers, real, and complex numbers. They can perform elementary calculations in these number systems.

  The first three topics merely have to be recalled. Then, complex numbers will be defined, and elementary operations will be exercized on the weekly worksheets, but not in the exam.

- systems of linear equations

  The students can define systems of linear equations, and they can solve these by applying the algorithm of Gauß elimination to them.

  These are the main *concrete* objects of linear algebra. These are defined and then, the method of Gauß elimination will be explained as one of the main tools to solve such equations. This algorithm will be exercised over and over in the weekly worksheets and will occur in the exam.

- vector spaces and linear maps

  The students can define vector spaces and linear maps. They can translate between these abstract notions and systems of linear equations.

  These are the main *abstract* objects of linear algebra. They formalize and abstract the idea of systems of linear equations. These notions simply have to be learnt by heart, but will not be assessed for the reasons explained at the end of Section 2.2.1.

- linear (in)dependence, bases, matrices, rank, product, inverse

  The students can define linear (in)depence and bases. Given a linear map, they can determine its matrix in a given basis, can compute the rank. They can compute products and inverses of matrices. Moreover, they can link the abstract definitions to concrete computations.

  These connect the previous two points, form a sort of dictionary between them, and are thus of central to this lecture. Many exercises in the weekly worksheets are meant to get used to these concepts, to illustrate this dictionary, and to compare both sides.

- determinant, characteristic polynomial, eigenvalues, normal forms

  The students can compute determinants using Gauß elimination, as well as using the Laplace formula. They can compute the characteristic polynomial and eigenvalues of a given matrix. From this, they can determine normal forms of matrices.

  These concepts are meant to simplify and classify linear maps. The theory is developed in the lecture, as are algorithms to compute them. These algorithms are applied to examples on the worksheets and in the exam.

- scalar products, norms, orthogonality

  The students can define and compute scalar products and norms of given vectors. Given a set of vectors, they can perform the Gram-Schmidt orthogonalization algorithm to obtain an orthonormal set of vectors.

  These introduce geometry to linear algebra and the Gram-Schmidt algorithm allows explicit computations. Especially this algorithm will be exercized on the worksheets and will occur in the exam.

exam: The exam is a written exam at the end of the term, and it lasts 90 minutes. There is a second exam (resit), also written and lasting 90 minutes, just before the beginning of next term. The problems in both exams are loosely based on the problems on the exercise sheets, but, of course, easier.

communication and the student's rôles: As common in mathematics, I mostly present definitions, results, proofs, and examples in the lecture at the blackboard. Due to the size of the audience (I had 465 students when teaching this course in SS 2014), an interaction with the students during lectures was almost impossible. However, in the exercise classes, the students could present their solutions to the homework, ask questions to the tutors, and comment on my lecture. Thus, via the tutors, I learnt about the student's success in the homework, and via their feedback in the exercise classes, I got an indirect feedback.

second impression: To give a second impression of a first-year lecture, I included in Section 3.2.1 a sample from my lecture notes from a first-year lecture at Düsseldorf University on linear algebra.

2.2.3 Example 2: Elliptic Curves

title	Elliptic Curves
audience	Master students or advanced Bachelor students
format	one semester with 4 hours of lecture and 2 hours of example classes per week
credits	9 ECTS points
language	English or German
assessment	weekly worksheets and either oral exams or a written exam (90 minutes) at the end of term

audience: This lecture is an introduction to algebraic geometry illustrated with elliptic curves. It is aimed at Master students and advanced Bachelor students, who already attended lectures in linear algebra and algebra and who might consider writing a thesis or even conduct research in this area. In particular, this is a smaller lecture (I had 16 students when teaching this course in SS 2016), I expected students to have different backgrounds, knowledge, but I also expected them to be rather high motivated. With this in mind, I wanted this lecture to be more on the creative side, with outlooks to other fields, and already giving the students first hints of modern research. After the lecture, it was possible to write a Bachelor or Master thesis with me, and I had recruiting students in mind, possibly even Ph.D. students in the long run.

learning outcomes and their assessment:

- affine and projective varieties

  The students know the definitions of affine and projective space, of algebraic sets and varieties, as well as the morphisms between them.

  These are the basic notions of algebraic geometry defined at the beginning. These notions simply have to be learnt by heart, but will not be assessed for the reasons explained at the end of Section 2.2.1.

- plane algebraic curves, singularities, differentials, and the genus

  The students know the definitions of singularities and differentials. For plane algebraic curves, they can compute the singular locus and the genus. They can translate between the abstract notion and the concrete computation.

  These illustrate the previously introduced abstract notions with explicit examples. Differentials, singularities and genera are introduced and their computation as well as the connection among them is explained and proven in the lecture. These computations are exercized in the worksheets, and will occur in the exam.

- elliptic curves, Weierstraß equations, discriminant, j-invariant

  The students can compute the discriminant and the j-invariant of an elliptic curve that is given by a Weierstraß equation. They can translate between the abstract definitions and the concrete computations.

Elliptic curves are the *abstract* topic of the lecture, and Weierstraß equations are the *concrete* realization in terms of equations. The remaining two notions are basic invariants of elliptic curves, which the students are expected to know to to compute for a given Weierstraß equation.

- the group law, heights and Mordell's theorem

  The student can perform elementary computations in the group law of an elliptic curve that is given by a Weierstraß equation. They can define and compute heights and they can state Mordell's theorem.

  The first is the main feature of elliptic curves, which takes a considerable amount of time to introduce and prove. Students are expected to learn important aspects of algebraic geometry from this. To apply this in explicit situations, height functions are introduced and heights will be computed on worksheets and in the exam. Mordell's theorem is a central in this field and its proof should be understood by the students. If time permits, I will give an outlook to Mordell's conjecture, now Faltings' theorem, and modern arithmetic geometry.

- elliptic curves over finite fields, Hasse's estimates, and the Weil conjectures

  The students know Hasse's estimates for elliptic curves over finite fields, can set up zeta functions, and know the statement of the Weil conjectures.

  This leads to more abstract algebraic geometry and to research questions. Here, proofs may be sketchy or incomplete, and the lecture will contain black boxes. Some trivial cases and aspects can be checked in the worksheets to give students an impression of the depth and difficulty of the material. I will not assess this.

- elliptic curve cryptography?

  The students know where elliptic curves enter modern cryptography.

  If time permits, connections to modern cryptography can be explained and discussed. I will not assess this in the worksheets or the exam. See also the outlook at the end of Section 3.4.

For the full and official module description, I refer to Section 3.4.

exam: Depending on the size of the audience, there will be either written exams (as explained in Section 2.2.2) or oral exams of 30 minutes.

communication and the student's rôles: During such a small lecture (I had 16 students when teaching this course in SS 2016), I oftentimes asked small comprehension questions. Also, in the exercise classes, the students could present their solutions to the homework, ask questions to the tutor, and comment on my lecture. Thus, I had several types of feedback: during the lecture, via my tutor, and, of course, from the evaluation.

second impression: To give a second impression from an advanced lecture, I included in Section 3.2.2 a sample from my lecture notes from a graduate lecture at Stanford Univeristy on algebraic surfaces, which was on an even higher level.

2.3 Additional Teaching Activities

For general audience and interregional

Hodge Conjecture	2022	Münchner Residenz
jointly organized with G. Kemper and J. Richter-Gebert		

The *Millenium Problems* are a list of seven problems and conjectures in mathematics that are central for mathematics of the 21$^{\text{st}}$ century. Already in 2020, the Junge Akademie der Leopoldina initiated a series of seven events all over Germany, *Die sieben größten Abenteuer der Mathematik*, dedicated to one problem each. Due to the Corona Pandemic, this series was postponed to 2022. These events were supported by the Deutsche Mathematiker Vereinigung (DMV) and the Deutsche Forschungsgemeinschaft (DFG) and under patronage of Bettina Stark-Watzinger, the federal minister of education and research of Germany.

Munich got the Hodge conjecture and the event, which was originally planned jointly with the LMU, took place on September 22, 2022 in the Bavarian Academy of Sciences in the Münchner Residenz. Together with Gregor Kemper (TUM) and Jürgen Richter-Gebert (TUM), I organised this evening. Approximately 200 people attended this evening, that is, listened to the two talks and watched the exhibition with mathematical models. I gave a lecture one of these lectures, namely an introduction to the Hodge conjecture.

For high school students, students, and almost general audience

Wechselläuten	2019	TUM
Über das Unendliche	2016	TUM
Riemanns Vermutung	2015	TUM
Komplexe Zahlen, Quaternionen und endliche Körper	2014	TUM

The first event was a Christmas Lecture at the Department of Mathematics on December 20, 2019, aimed at students of all semesters and general audience, where I introduced Change Ringing, that is, the English art of ringing church bells, with audio samples, live ringing of hand bells, and the connections of this art to algebra.

The second event was short talk aimed at students and their parents at the Fakultätstag Mathematik on June 10, 2016, where I explained how to count up to infinity and beyond.

The third event was a one-day event for high school students and their parents, the Tag der Forschung on March 7, 2015, where I gave a talk on the famous Riemann conjecture as an example of research in pure mathematics.

The fourth event was a one-day event for high school students, the Schülertag on February 6, 2014, where first, I gave an introductory talk to algebra, and then, I supervised a workshop for high school students, where they could apply the just-learnt material.

For undergraduate students

Relativistic Addition and Group Theory	2010	Stanford

This was a talk aimed at undergraduate students to give an application of abstract concepts from pure mathematics (algebra) to physics (special relativity) within the Stanford University Mathematical Organization (SUMO) speaker series for undergraduates on November 17, 2010.

Kleine AG "Algebraische Geometrie und Zahlentheorie"

Alterations following de Jong (jointly organized with Kay Rülling)	2007	Bonn
Simultaneous Resolution of Singularities (jointly organized with Oliver Lorscheid)	2007	Bonn
Classification of holonomy groups (jointly organized with Martin Möller)	2006	Bonn
Equivalences of derived categories of coherent sheaves (jointly organized with Arend Bayer)	2004	Bonn
Elliptic curves with complex multiplication (jointly organized with Inken Vollaard)	2003	Köln

This is a regular one-day workshop between the universities of Bonn, Köln, Düsseldorf, Essen-Duisburg, Heidelberg, Mainz, and others taking place twice per semester. It is aimed at Ph.D. students and postdoctorial students, and every time, a recent research topic in algebraic geometry, complex geometry, or number theory is addressed.

Lecture series at summer schools

Models and degenerations of varieties	2018	Łukęcin (Poland)
Crystalline cohomology and period domains	2016	Luminy (France)
Models of curves, Abelian varieties, and K3 surfaces	2016	Nesin math village (Turkey)
K3 and Enriques surfaces	2016	Moscow
Unirationality and rational curves	2014	Moscow
The crystalline Torelli theorem	2013	Strasbourg
Algebraic surfaces in positive characteristic	2009	Sogang (Korea)

These were lecture series, typically 3-4 lectures each, I gave at summer schools, conferences, and universities. These were aimed at Ph.D. students, postdoctorial students, as well as researchers. I even turned two of these lecture series into lecture notes, which have been published meanwhile.

Lectures on Supersingular K3 Surfaces and the Crystalline Torelli Theorem,
K3 Surfaces and Their Moduli, Progress in Mathematics 315,
Birkhäuser (2016), 171-235 (64 pages).

Algebraic Surfaces in Positive Characteristic,
Birational Geometry, Rational Curves, and Arithmetic, Simons Symposia,
Springer (2013), 229-292 (63 pages).

Summer Schools

Elliptische Kurven und die Vermutung von Birch und Swinnerton-Dyer	2017	St. Johann (South Tyrol)

This was a two week summer school for advanced Master students and Ph.D. students, who hold a stipend from the German National Academic Foundation (Studienstiftung des deutschen Volkes). The course introduced the classical topic of elliptic curves and led to the BSD conjecture, which is one of the seven Clay Millennium Problems and thus, one of the leading questions in contemporary mathematics. This was organized jointly with Thomas Geisser from Tokyo.

Conferences

Periods, Motives, and Differential Equations jointly organized with B. Chiarellotto, L. Di Vizio, E. Lepage	2022	Paris
Moduli Day jointly organized with F. Gounelas	2020	Munich
Crystals and Geometry in Characteristic p jointly organized with O. Gregory, R. Laface, and G. Martin	2018	Munich
Geometry and Arithmetic of Surfaces jointly organized with C. Frei and U. Derenthal	2014	Munich
Western Algebraic Geometry Symposium (WAGS) jointly organized with D. Erman, J. Hall, J. Li, and R. Vakil	2011	Stanford

The first was planned as a conference with 16 invited speakers from Europe and the USA at the Institut Henri Poincaré (IHP) in Paris. It was supposed to take place in April 2020, but due to the Corona Pandemic, we had to cancel it on very short notice. First, we postponed to 2021 and now, to April 11-15, 2022.

The second was a one-day workshop with 4 invited speakers from Europe and approximately 20 participants and took place at TUM on January 30, 2020.

The third was a conference with 12 invited speakers from Europe, Japan and the USA and approximately 40 participants and took place at TUM, April 4-6, 2018.

The fourth was a conference with 17 invited speakers from Europe, Japan, and the USA and approximately 40 participants that took place at LMU and TUM, March 17-21, 2014.

The fifth was a workshop with 6 invited speakers from the USA and approximately 30 participants that took place at Stanford University, April 8-10, 2011.

Colloquium series

Munich Summer Colloquium jointly organized with R. Frank, G. Friesecke, G. Kutyniok	2022	Munich

In order to better connect the two main universities in Munich, LMU and TUM, the mathematics departments decided to create an annual and prestigious colloquium, which should take place every summer. In this colloquium, two internationally recognised speakers should give talks, where one speaker should be more from the pure side of mathematics and the other speaker from the more applied side. The organising team consisted of Rupert Frank (LMU), Gero Friesecke (TUM), Gitta Kutyniok (LMU), and myself.

The first event of this sort took place on June 2, 2022 in the Siemens Foundation in the Nymphenburger Schloß in Munich. The two speakers were Peter Scholze (Bonn) and Weinan E (Princeton/Beijing).

Book

Jointly with Igor Dolgachev (Ann Arbor), François Cossec, and Shigeyuki Kondō (Nagoya), I am currently writing a compendium on Enriques surfaces. This is a specialized topic in algebraic geometry, and the two volumes are aimed at researchers and advanced Ph.D. students. They will appear soon, hopefully in 2024.

2.4 Further Education

TUM Tenure Track Teaching Modules

T2	Implementing Concepts of Teaching and Learning	2016
T1	Concepts of Teaching	2015
R2	Examining and supervising junior researchers	2014
R1	Safeguarding the rules of scientific practice and enhancing your visibility as a researcher	2014

The first two two seminars, organized by the ProLehre team, both taking place at TUM, the first February 29-March 1, 2016 and the second June 29-30, 2015, to introduce young professors to concepts of teaching, the design of lectures, constructive alignment, evaluations, and the supervision of students.

The final two seminars both took place at TUM on December 12, 2014 and December 10, 2014, respectively. Although these were not focused on teaching, there were units on supervision and good scientific practice, both of which are relevant to teaching, which is why I included them here.

Advanced Training

Presentation training for grant applications	2015
Management seminar	2015
Seminar on argumentation techniques	2015

The first was a training for a grant application presentation for the ERC in Brussels organized by TUM ForTe and Pierre Freimüller at TUM on September 16, 2015 including videotaping and discussing my presentation.

The second was a management seminar with the title “Führend Wissen schaffen - Führungsstil und Führungskompetenz” to learn basic concepts of leadership, management, and supervision organized by the TUM school of management at TUM on September 18, 2015.

The third was a seminar with the title “Kunstgriffe der Argumentation - Schlagfertig im Lehr- und Wissenschaftsbetrieb” organized by TUM ProLehre at TUM on January 14, 2015 to learn about basic argumentation techniques, which comes in particularly handy in faculty meetings.

Exchange and Feedback

ProLehre Forum für ProfessorInnen	2015
Visit of TUM ProLehre in my lecture	2013

The first is a platform between TUM ProLehre and young professors to exchange teaching experiences, problems, and improvements in every day teaching. Our first meeting was on November 24, 2015.

The second was a visit of TUM ProLehre in my lecture “Algebraische Geometrie” on November 25, 2013, followed by feedback and discussion.

2.5 Evaluations and Feedback

General remarks

I like my teaching - as well as my research - to be open, experimental, and with a certain extent of searching, with detours, and also taking the quest for knowledge rather than only achievements into account - see also Schönberg's quotation at the beginning of Section 1.1, which I like a lot. However, since quests may go astray, it is only natural that feedbacks - be it comments by colleagues, be it official evaluations by students - are a part of teaching needed to constantly calibrate and improve. Therefore, I view all feedback as an integral part of my teaching that I take seriously and that I always reflect. However, I may not always follow all the opinions expressed, since I do not think that evaluations are laws to abide by.

Evaluations - quantitative results

In 2014, I taught a service lecture on linear algebra for computer scientists. Since 119 students participated in the evaluation, this evaluation is statistically more significant than some of the smaller lectures I taught.

Design	More than 90% of the students indicated that the learning outcomes were clear and that the topics were well ordered and harmonized.
Explanations	More than 70% indicated that they followed well my explanations. More than 85% indicated that they could at least follow reasonably well.
Time frame	More than 80% indicated that the time frames for the topics covered were adequate.
Examples	More than 70% found the examples I gave vivid (note: this was a lecture on fundamental mathematics).
Media	Here, I refer to the discussion below.
Atmosphere	Around 90% indicated that I responded to their questions and their suggestions. Around 80% found my explanations comprehensible and more than 90% had the impression that I was always well-prepared. Finally, around 80% attested me interest in the students' success.
Level	Around 60% found the level of difficulty just right and another 30% found it slightly too difficult. Around 65% found my speed just right and another 30% found it slightly too high. Almost 70% found the amount of material just right, and another 25% found it slightly too much.
Abilities	Around 85% had the impression that they could name the important notions and concepts of the lecture, more than 80% said they could give an overview over the lecture, and around 80% were confident that they could answer a typical question from the subject of the lecture.
Overall	Almost 90% had a good overall impression of this lecture and almost 95% were at least satisfied with it.

Evaluations - qualitative results

In 2013 and 2015, I taught the Master course "Algebra 2" for students in their fourth or sixth semester. In both lectures, 13 students participated in the evaluation, the lecture was far more personal than the previously discussed one, and the comments were less generic.

The students' comments attested me enthusiasm, an informal style, interesting material, and that I pointed out connections to other fields in and outside mathematics. Some even wrote I had an entertaining style, which I find double-edged when it comes to teaching. I also had positive comments concerning answering questions during lectures and clarifying confusions immediately. I had no official script, but it was met with praise that I scanned and uploaded my handwritten notes to the website of the lecture (see Section 3.2.2 for an example).

Finally, some students complained that the speed and density of the material was too low in the first four weeks and that it was too high towards the end. In fact, I planned the course to start slower, since students had a different background and I wanted catching up to be possible, and then, I wanted to increase in speed and depth. However, it is perfectly possible that this course needs a little bit of fine-tuning there.

The usage of slides - a deliberate decision

Let me comment on a point from the students' feedback, which I consider of some importance concerning my Teaching Concepts. Namely, several students complained in the evaluations that I should use slides or other new media rather than the classical and old school chalk-and-blackboard lecture, see Section 2.2.1. However, not using slides or powerpoint was a deliberate decision.

1. First, one argument in favor of slides is that the lecturer can upload them online and make them available to the students. But since I already provided students with a script (see discussion above), nobody had to take notes, and I believe that the script was more detailed than slides would have been.

2. Second, I do not believe that slides improve the lecture - quite the contrary! I believe that mathematics should be developed on a blackboard rather than being spoon-fed in the form of prepared slides that are merely read out by the lecturer. I believe that slides also have the disadvantage that the lecturer is less flexible: I can do less detours and I am less spontaneous. Also, since everyone in the audience, including the lecturer, would be fixed on a screen, the interaction between the audience and the lecturer are more limited than a lecture of such a big size is anyway. Finally, most lectures using slides simply present far too much material and at a far to high speed!

Further feedback

In 2013, I took the opportunity of having TUM ProLehre visit my lecture "Algebraische Geometrie", including feedback and discussion afterwards (see Section 2.4). What I took out of this was to frame my lectures better, that is, to start a lecture by recalling where we stopped last time, maybe repeat some of the important concepts or notions, where we will go on today, and where we are heading to (see also Section 2.2.1). These small surveys are not very time consuming and help a lot.

In 2015 and 2016, I attended the TUM Tenure Track Teaching Modules T1 and T2, which were about abstract concepts of teaching, as well as practical implementations, see Section 2.4. Since Fall 2015, I also participated in an open and informal discussion group, organized by TUM ProLehre, with other professors from the mathematics department that meets once or twice per semester to discuss teaching-related questions.

Concluding remarks

Quite generally, I was happy with the results of this evaluation and I believe that they show that I am on the right track. The main impact that evaluations, feedback talks, seminars, etc. had on me, were the following: be it service lectures, be it master lectures, be it the supervision of Ph.D. students, I now tend to set up more rules, mutual agreements, and a frame first. This gives an easier start and much more grip, especially for weaker students or for students, who are, for some reason or another, not so much interested in the subject at hand. On the other hand, with good and promising students, it is always easy to relax these rules and to give them more freedom as lectures, seminars, and supervisions progress.

2.6 Teaching Awards

Goldener Zirkel

Every semester, the Fachschaft Mathematik (students' union of the department of mathematics) awards the *Goldener Zirkel* (Golden Compass) for the best lectures in the categories Grundlagenvorlesung (basic lecture), Vertiefungsvorlesung (specialized lecture), and Übungsbetrieb (example classes). So far, I was awarded this twice for the following lectures

WS 2016/17	Algebra 1 Category: Vertiefungsvorlesung
WS 2019/20	Lineare Algebra 1 Category: Grundlagenvorlesung

Moreover, I received the third prize in these awards for

SS 2022	Algebraic Geometry Category: Vertiefungsvorlesung

3 Appendix

3.1 Evaluations

In an earlier version of this teaching portofolio, I included the evaluations of all lectures. In 2022, these evaluations took up more than 100 pages (even after removing the individual comments of the students), which is why I decided not to include the evaluations any more. They are still available upon request.

3.2 Excerpts from scripts

In the sequel, I give two samples from scripts to illustrate some of the abstract conceptions presented above, as well as to give an impression of my teaching. The first sample is from a large lecture for first year students of computer science and the second sample is from an advanced graduate course.

3.2.1 Example 1: Lineare Algebra für Mathematiker

This is a sample of the lecture notes of my lecture on linear algebra, held at the HHU Düsseldorf in the Sommersemester 2009. The lecture was aimed at students, who had mathematics as their second subject (mainly physicists and computer scientists), as well as students in mathematics, who had already failed their exam once. Therefore, the audience was not very motivated on average. See Section 3.3 for one of the worksheets of this lecture.

The excerpt are the notes of a 90 minute lecture: it starts with an important definition, then I gave some elementary and not so elementary examples, and showed that this notion is stable under intersections. See Section 2.2.1 for the general style of such lectures and to Section 2.2.2 for the placing of this content within the whole lecture.

<u>Definition</u>: Sei V ein K-Vektorraum. Eine Teilmenge $U \subseteq V$ heißt <u>Untervektorraum</u> falls

(i) $x, y \in U \Rightarrow x + y \in U$
(ii) $\lambda \in K,\ x \in U \Rightarrow \lambda \cdot x \in U$
(iii) $0 \in U$.

D.h. $0 \in U$ und U ist abgeschlossen unter Vektoraddition und Skalarmultiplikation.

<u>Bspl 1</u>: Sei $\sum_{j=1}^{n} a_{ij} x_j = 0 \quad \forall\ 1 \le i \le m$ ein LGS. Dann ist die Lösungsmenge

$$U = \{ (\lambda_1, \dots, \lambda_n) \mid \sum_{j=1}^{n} a_{ij} \lambda_j = 0,\ 1 \le i \le m \} \subseteq K^r$$

ein Untervektorraum.

<u>Bspl 2</u>: V ein K-Vektorraum. Dann sind ~~~~

(1) $U = V$
(2) $U = \{0\}$

Untervektorräume von V.

<u>Bspl. 3</u>: Sei $V := \{ f : \mathbb{R} \to \mathbb{R} \}$ der $\mathbb{R}$-Vektorraum aller Abbildungen von $\mathbb{R}$ nach $\mathbb{R}$. Dann ist

$$U := \mathcal{C}(\mathbb{R}) := \{ f : \mathbb{R} \to \mathbb{R} \quad f \text{ ist stetig} \} \subseteq V$$

ein Untervektorraum.

Dahinter stecken folgende Sätze aus der Analysis:

(1) f, g stetig $\Rightarrow$ $f + g$ stetig
(2) $\lambda \in \mathbb{R}$, f stetig $\Rightarrow$ $\lambda \cdot f$ stetig
(3) $f(x) = 0 \ \forall x \in \mathbb{R}$ ist stetig.

Bspl 4 Sei L ein Körper. Eine Teilmenge $K \subseteq L$ heißt *Unterkörper*, wenn gilt

(i) $x, y \in K \Rightarrow x+y \in K, \; x \cdot y \in K$

(ii) $x \in K \Rightarrow -x \in K$

(iii) $x \in K, x \neq 0 \Rightarrow x^{-1} \in K$

(iv) $0, 1 \in K$

Dann werden die Verknüpfungen $+, \cdot$ von L zu Verknüpfungen auf K (nach (i)) und (ii) - (iv) stellen sicher, daß K mit diesen Verknüpfungen zu einem Körper wird.

Insbesondere ist L ein K-Vektorraum und $K \subseteq L$ ist ein Untervektorraum, z.B. $\mathbb{Q} \subseteq \mathbb{C}$ oder $\mathbb{R} \subseteq \mathbb{C}$.

Ist U ~~ein~~ $\subseteq V$ ein Untervektorraum, so sagt man, daß die Verknüpfungen $+: V \times V \to V$ und $\cdot: K \times V \to V$ die Verknüpfungen $+: U \times U \to U$ und $\cdot: K \times U \to U$ *induzieren*.

Satz: Die Verknüpfungen machen U zu einem K-Vektorraum

Bew.:
- U ist Gruppe: neutrales Element $0 \in U$
  Inverses: $x \in U \Rightarrow -x \in U$
  da $-x = (-1) \cdot x$.
- es gibt eine Verknüpfung
  $K \times U \to U$
- die Bedingungen an einen Vektorraum vererben sich von V auf U.

$\square$

Wir haben folgendes Kriterium für Untervektorräume

<u>Lemma</u> : Sei V ein K-Vektorraum. Eine Teilmenge $\mathcal{U} \subseteq V$ ist genau dann Untervektorraum, wenn

(i) $\mathcal{U} \neq \emptyset$

(ii) $x, y \in \mathcal{U}, \lambda \in K \Rightarrow x + \lambda \cdot y \in \mathcal{U}$

gilt.

<u>Bew.</u>: „$\Rightarrow$" trivial

„$\Leftarrow$" seien $x, y \in \mathcal{U}$. Für $\lambda = 1 \in K$ folgt

$$x + y = x + 1 \cdot y \in \mathcal{U}$$

sei $y \in \mathcal{U}$. Für $x = 0$ folgt

$$\lambda \cdot y = 0 + \lambda y = x + \lambda y \in \mathcal{U}$$

da $\mathcal{U} \neq \emptyset$ existiert ein $x \in \mathcal{U}$ und mit $\lambda = -1$ erhalten wir

$$0 = x + (-x) = x + (-1) \cdot x \in \mathcal{U}$$

□

Durchschnitte von Untervektorräumen liefern wieder Untervektorräume:

<u>Satz</u> : Sei V ein K-Vektorraum und $\mathcal{U}_\alpha \subseteq V$, $\alpha \in I$ eine Familie von Untervektorräumen. Dann ist

$$\mathcal{U} := \bigcap_{\alpha \in I} \mathcal{U}_\alpha \subseteq V$$

ein Untervektorraum.

<u>Bew.</u>: - aus $0 \in \mathcal{U}_\alpha \ \forall \alpha \in I$ folgt $0 \in \mathcal{U}$, also $\mathcal{U} \neq \emptyset$

- seien $x, y \in \mathcal{U}$ und $\lambda \in K$. Dann sind $x, y \in \mathcal{U}_\alpha \ \forall \alpha \in I$ und somit $x + \lambda y \in \mathcal{U}_\alpha \ \forall \alpha \in I$ also $x + \lambda y \in \mathcal{U}$

aus dem Lemma folgt dann, daß $\mathcal{U}$ Untervektorraum ist.

□

WARNUNG: Die Vereinigung $\bigcup_{\alpha \in I} U_\alpha \subseteq V$ ist im allgemeinen kein Untervektorraum

Wir benötigen daher den folgenden Begriff:

Definition: Ein Vektor $x \in V$ heißt Linearkombination aus den Vektoren $x_1, \dots, x_n \in V$, wenn es $\lambda_1, \dots, \lambda_n \in K$ gibt, so daß

$$x = \lambda_1 x_1 + \dots + \lambda_n x_n$$

gilt.

Notation:

$$\langle x_1, \dots, x_n \rangle := \{ x \in V \mid x \text{ ist Linearkombination aus } x_1, \dots, x_n \}$$
$$= \{ \textstyle\sum_{i=1}^n \lambda_i x_i \mid \lambda_1, \dots, \lambda_n \in K \}$$
$$\subseteq V$$

Satz: Die Teilmenge $\langle x_1, \dots, x_n \rangle$ ist ein Untervektorraum.

Bew.: Sei $U = \langle x_1, \dots, x_n \rangle$

- setzt man $\lambda_1 = \dots = \lambda_n = 0$ so folgt
$$0 = 0 \cdot x_1 + \dots + 0 x_n \in U.$$

~~sei $x = \sum \lambda_i x_i$, $y = \sum \nu_i y_i$ und $\lambda \in K$~~

- sei $x, y \in U$ und $\lambda \in K$. Also gibt es μ_i, ν_i $1 \le i \le n$ so daß
$$x = \sum_{i=1}^n \mu_i x_i$$
$$y = \sum_{i=1}^n \nu_i x_i$$

dann ist
$$x + \lambda \cdot y = \sum^n \mu_i x_i + \lambda \cdot \sum^n \nu_i x_i$$
$$= \sum_{i=1}^n \underbrace{(\mu_i + \lambda \cdot \nu_i)}_{\in K} \cdot x_i$$
$$\in U.$$

nach Lemma ist U somit Untervektorraum. $\square$

3.2.2 Example 2: Algebraic Surfaces

This is a sample of the lecture notes of my lecture on algebraic surfaces, held at Stanford University in the spring term 2011. This was a course aimed at graduate students and had the goal of introducing a topic at research level. In particular, the speed was high and the level was very advanced.

The excerpt are the notes of a 90 minute lecture: I taught from these notes, filled in the details in the lecture, and uploaded the notes after the lecture on my webpage. Since the students were graduate students, these notes were rather dense and leaving out many details. They were meant as a survey of the most important ideas and concepts of the topic, rather than a detailed book. However, I also recommended a proper textbook, too long to possibly teach from, but together with the lecture, students could delve arbitrarily deep into the subject.

Remarks

1) if $C \cong \mathbb{P}^1$, $C^2 < 0$ there always exists a contraction $X \xrightarrow{f} X_0$ of X_0, in gen'l X_0 will not be smooth

2) if C is an irred. curve with $C^2 < 0$ there always exists a contraction in the category of algebraic spaces, but maybe not in cat. of schemes

Definition A surface (smooth, projective) is called minimal, if it does not contain (-1)-curves, i.e. curves E with

$$E^2 = K_X E = -1 \quad , \quad E \cong \mathbb{P}^1_k$$

Definition: If X, X' are smooth projective and birat'l surfaces and X' is minimal then X' is called a minimal model of X.

<u>Thm</u> (Existence of minimal models)

Let X be a smooth projective surface $/k=\bar{k}$

Then $\exists$ sequence of blow-downs

$$X \xrightarrow{f} X_0$$

s.th. X_0 is minimal.

<u>proof</u>: if X is not minimal then $\exists$ (-1) curve $E \subseteq X$

by Castelnuovo's theorem, there exists

$$X \longrightarrow X_1$$

a blow-down of E.

if X_1 is not minimal : Continue!

Since $NS(X) = NS(X_1) \oplus \mathbb{Z} \cdot E$

we see

$$rk(NS(X)) = rk(NS(X_1)) + 1$$

ie. this process of blow-downs must stop. $\square$

<u>Remark</u> From

$$\mathbb{P}^2 \underset{\substack{\text{open}\\ \text{dense}}}{\supseteq} \mathbb{A}^2 \cong \mathbb{A}^1 \times \mathbb{A}^1 \underset{\substack{\text{open}\\ \text{dense}}}{\subseteq} \mathbb{P}^1 \times \mathbb{P}^1$$

we see that $\mathbb{P}^2$ and $\mathbb{P}^1 \times \mathbb{P}^1$ are birational

Also, they are minimal (excercise!)

$\Rightarrow$ minimal models are <u>not</u> unique.

3.3 Assessments and Worksheets

3.3.1 Example 1: Lineare Algebra für Mathematiker

This is a worksheet from my lecture on linear algebra, held at the HHU Düsseldorf in the Sommersemester 2009. This lecture was aimed at students, who had mathematics as their second subject (mainly physicists and computer scientists), as well as students in mathematics, who had already failed their exam once. Therefore, the audience was not very motivated on average.

In the worksheet below, the problems are ordered according to difficulty: problem 1 exercises an algorithm given in the lecture and I mentioned several times that computations similar to these are highly relevant for the exam. Problems 2 and 3 are more abstract and finally, problem 4 contains an application of the lecture's material to computer science - hoping to give the students' motivation a little nudge...

Dr. Christian Liedtke　　　　　　　　Sommersemester 2009
Dipl. math. Christian Löffelsend

Übungen zur Linearen Algebra I

Blatt 11
(Inverse)

Aufgabe 1. Invertieren Sie die folgenden Matrizen mit dem Algorithmus aus der Vorlesung!

(i) $\begin{pmatrix} 9 & 5 \\ 7 & 4 \end{pmatrix} \in \mathrm{Mat}(2,\mathbb{R})$　　(ii) $\begin{pmatrix} 1 & 3 \\ 3 & 2 \end{pmatrix} \in \mathrm{Mat}(2,\mathbb{F}_5)$

(iii) $\begin{pmatrix} 1 & 2 & 0 \\ 2 & 3 & 1 \\ 4 & 5 & 2 \end{pmatrix} \in \mathrm{Mat}(3,\mathbb{Q})$　　(iv) $\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 2 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix} \in \mathrm{Mat}(4,\mathbb{F}_3)$

Aufgabe 2. Sei K ein Körper. Zeigen Sie, dass die Abbildung

$$\begin{array}{rccc} \varphi & : & \mathrm{GL}(2,K) & \to & K^{\times} \\ & & \begin{pmatrix} a & b \\ c & d \end{pmatrix} & \mapsto & ad-bc \end{array}$$

ein Gruppen-Homomorphismus ist!

Aufgabe 3. Sei $A = (a_{ij}) \in \mathrm{Mat}(n, \mathbb{R})$ und es gelte

$$\sum_{j=1, j\neq i}^{n} |a_{ij}| < |a_{ii}|, \qquad \text{für } i = 1, ..., n.$$

Zeigen Sie, dass A invertierbar ist! Stimmt das immer noch, wenn man nur "$\leq$" statt "$<$" verlangt?

Aufgabe 4. (Kryptographie) Sei A das Alphabet, das nur aus Kleinbuchstaben, sowie den Zahlen 2,3,4 besteht. Zum Verschlüsseln einer Nachricht N, die in dem Alphabet A geschrieben ist, werde der folgende Algorithmus benutzt:

1. Sei $N = (n_1, n_2, ...) \in A^{3m}$ die Nachricht.
2. Teile die Nachricht in Dreitupel auf: $w_1 := (n_1, n_2, n_3)$, $w_2 := (n_4, n_5, n_6)$,... d.h. jedes w_i ist ein Element in A^3.
3. Benutze die Tabelle unten, um jedes dieser Tupel in einen Spaltenvektor $v_i \in \mathrm{Mat}_{3\times 1}(\mathbb{F}_{29})$ umzuwandeln.
4. Multipliziere jeden dieser Spaltenvektoren v_i von links mit der Matrix
$$A := \begin{pmatrix} 1 & 1 & 1 \\ 0 & 3 & 1 \\ 0 & 5 & 2 \end{pmatrix} \in \mathrm{Mat}(3, \mathbb{F}_{29})$$
5. Wandele die Ergebnisvektoren Av_i mit Hilfe der Tabelle wieder in Drei-Tupel $w_i' \in A^3$ um und schreibe diese als neue verschlüsselte Nachricht N' auf.

Dieses wurde nun auf eine Nachricht $N \in A^{3\cdot 8}$ angewendet. Das Ergebnis lautet wie folgt:

$$N' = (b, u, 2), (2, x, e), (t, r, f), (e, d, b), (p, p, 2), (3, 2, p), (n, s, b), (3, o, k)$$

Bestimmen Sie die ursprüngliche Nachricht!

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p
16	17	18	19	20	21	22	23	24	25	26	27	28			
q	r	s	t	u	v	w	x	y	z	2	3	4			

BEMERKUNG: Dieser Algorithmus wurde tatsächlich zum Verschlüsseln von Festplatten benutzt, hat allerdings eine entscheidende Schwäche, siehe

http://www.heise.de/security/Billige-Verschluesselung-knacken--/artikel/122347

Abgabe: Bis Mittwoch, den 1.7.2009 um 11:10 Uhr in den Zettelkästen.

3.3.2 Example 2: Algebraic Geometry

This is a worksheet from my lecture on algebraic geometry, held at TUM in the Wintersemester 2015/16. This is an lecture at master level, aimed at advanced B.Sc. students, M.Sc. students, and I already had in mind looking for possible Ph.D. students.

Every now and then, I included mini-projects in these worksheets, which gave the students the possibility to see whether they would like to write a thesis in algebraic geometry. On the other hand, it also gave me first hints whether there are students interested and talented in this field. Of course, it was made clear to the students that these mini-projects are of a somewhat higher difficulty.

In the worksheet below, problems 1 and 2 are standard exercises, where the students merely have to apply definitions and which should be doable without too much effort. Problems 3 and 4 are such a mini-project, aimed at particularly interested students.

Prof. Dr. C. Liedtke
M.Sc. D. Boada

due Tuesday, December 22, 2015

Exercises Algebraic Geometry
Sheet 10

1. *Duality for locally free sheaves.* Let $(X, \mathcal{O}_X)$ be a ringed space, and $\mathcal{E}$ be a locally free $\mathcal{O}_X$-module of finite rank.

1. For any $\mathcal{O}_X$-module $\mathcal{F}$, we have
$$\mathcal{H}om_{\mathcal{O}_X}(\mathcal{E}, \mathcal{F}) \cong \mathcal{E}^\vee \otimes_{\mathcal{O}_X} \mathcal{F}.$$
2. For any $\mathcal{O}_X$-modules $\mathcal{F}$, $\mathcal{G}$, we have
$$\mathrm{Hom}_{\mathcal{O}_X}(\mathcal{E} \otimes_{\mathcal{O}_X} \mathcal{F}, \mathcal{G}) \cong \mathrm{Hom}_{\mathcal{O}_X}(\mathcal{F}, \mathcal{H}om_{\mathcal{O}_X}(\mathcal{E}, \mathcal{G})).$$

2. *Homomorphisms.* Let R be a ring.

1. Show that there are no non-trivial homomorphisms of group schemes $\mathbb{G}_{m,R} \to \mathbb{G}_{a,R}$ over $\mathrm{Spec}R$.
2. If R is reduced, show that there are no non-trivial homomorphisms of group schemes $\mathbb{G}_{a,R} \to \mathbb{G}_{m,R}$ over $\mathrm{Spec}R$.
3. For each $0 \neq r \in R$ with $r^2 = 0$, find a non-trivial homomorphism of group schemes $\mathbb{G}_{a,R} \to \mathbb{G}_{m,R}$ over $\mathrm{Spec}R$.

3.+4. *Group Schemes of Rank 2.* Let k be a ring. The objective of this exercise is to classify group schemes of the form $\mathrm{Spec}R \to \mathrm{Spec}k$, where R is a free k-module of rank 2. (If you want, you may also assume that k is a field.) Put a little bit sloppily, these are the group scheme analogs of "groups of order 2"...

1. If M is a free k-module of rank 2, and $e^\# : M \to k$ linear and surjective, show that $\ker(e^\#)$ is a free k-module of rank 1.
2. Let R be a Hopf algebra over k that is a free k-module of rank 2. Show that $I := \ker(e^\#) = kx$ (where $e : \mathrm{Spec}k \to \mathrm{Spec}R$ is the zero section) for some $x \in R$, and $\mu^\#(x) = x \otimes 1 + 1 \otimes x + bx \otimes x$ (where $\mu : \mathrm{Spec}R \times \mathrm{Spec}R \to \mathrm{Spec}R$ is the multiplication) for some $b \in k$.
3. Show that $x^2 + ax = 0$ for some $a \in k$, and deduce
$$R \;=\; k[x]/(x^2 + ax)$$
as a k-algebra.
4. Use $\mu^\#(x^2) = (\mu^\#(x))^2$ to show $(2 - ab)^2 = 2 - ab$.
5. Show that $\rho^\#(x) = cx$ (where $\rho : \mathrm{Spec}R \to \mathrm{Spec}R$ is the inversion) with $c^2 = 1$. Use this and $0 = (\rho^\#, \mathrm{id})\mu^\#(x)$ to show $c = 1$ and $ab = 2$.
6. Conversely, given $a, b \in k$ with $ab = 2$, then
$$R_{a,b} \;:=\; k[x]/(x^2 + ax)$$
with $\mu^\#(x) = x \otimes 1 + 1 \otimes x + bx \otimes x$ and $\rho^\#(x) = x$ turns $R_{a,b}$ into a Hopf algebra. We set
$$G_{a,b} \;:=\; \mathrm{Spec}R_{a,b} \;\to\; \mathrm{Spec}k\,.$$
7. Show that $G_{a,b}$ is isomorphic to $G_{a',b'}$ if and only if $a = ua'$ and $b = u^{-1}b'$ for some invertible element $u \in k$.
8. Describe $G_{1,2}$ and $G_{2,1}$. If $2 = 0$ in k, describe $G_{0,0}$.
9. Show that $G^D_{a,b} = G_{b,a}$.

Remark: The classification of group schemes $\mathrm{Spec}R \to \mathrm{Spec}k$, where R is a free k-module of rank p with $p > 0$ a prime number, is due to F. Oort and J. Tate, and the a, b in the previous exercise are called *Tate–Oort parameters* of the group scheme.

3.4 Module Descriptions

Module Description for Elliptic Curves (MA5114)

This is the module description for my lecture on elliptic curves, held for the first time at TUM in the Sommersemester 2016. This course was aimed at advanced Bachelor students, as well as Master students with a background in linear algebra and algebra.

The idea of this course is to give a lecture in advanced algebra, introducing an interesting classical and modern subject, and which is also an introduction to algebraic geometry. Therefore, it starts with introducing affine and projective varieties, and especially plane algebraic curves. This illustrates some abstract concepts from algebraic geometry, such as differentials and the genus. However, restricting these concepts to algebraic curves makes them explicit and computable. In fact, this is the general theme of the lecture: between modern abstract algebraic geometry and explicit applications to a particularly interesting class, namely elliptic curves.

Possible variations of this lecture in the future

If time permits, also applications to modern cryptography can be included in this lecture. Alternatively, I was thinking about accompanying this lecture by a seminar in elementary number theory and cryptography with the aim to introduce elliptic curve cryptography. By working out and extending their seminar talks, this would give students the possibilty of writing Bachelor theses.

Modulbezeichnung	Elliptische Kurven / Elliptic Curves
Modulniveau	Master
Semesterdauer	einsemestrig
Angebot	unregelmäßig
Sprache	englisch
ECTS	9
Beschreibung der Prüfungsleistung	The exam will be in written form (90 minutes). Students are allowed to bring a hand-written sheet of paper (A4, both sides), containing any information they like, but no further aids and resources. The students will be asked to perform explicit calculations with algebraic curves, such as determining their singularities and their genus. They will be asked to perform elementary computations with the group law of an elliptic curve in Weierstraß form. For elliptic curves over finite fields they might be asked to estimate the number of rational points. Finally, they will be asked to transfer results and techniques discussed in the lecture and the exercise classes to new examples.
Prüfungsart	Klausur (schriftlich)
Prüfungsdauer	90 Minuten
Wiederholungsmöglichkeiten	Semesterende
Voraussetzungen	MA 1101 Linear Algebra 1 MA 1102 Linear Algebra 2 MA 2101 Algebra
Inhalt	Affine and projective varieties, plane algebraic curves and singularities, differentials and genus of curves Weierstraß equations, discriminant, j-invariant, the group law, heights and Mordell's theorem, elliptic curves over finite fields, Hasse's estimates and the Weil conjectures
Angestrebte Lernergebnisse	At the end of this lecture, the students should know the definitions of affine and projective varieties, of elliptic curves, their basic invariants (discriminant, j-invariant), the theorems of Bézout and Mordell, and Hasse's bounds on rational points over finite fields. They should be able to compute differentials and the genus of plane curves. Moreover, the should be able to compute basic invariants and the group law of elliptic curves given in Weierstraß form.
Lehr- und Lernmethoden	Vorlesung, Übung, Hausaufgaben, Übungsaufgaben
Medienform	Tafelarbeit, Übungsblätter
Literatur	Knapp: *Elliptic Curves* Shafarevich: *Basic Algebraic Geometry* Silverman: *The Arithmetic of Elliptic Curves*
Modulverantwortlicher	Christian Liedtke
Lehrveranstaltungen	Vorlesung 4 SWS Übungen 2 SWS

Module Description for Algebraic Surfaces (MA5132)

This is the module description for my lecture on algebraic surfaces, held for the first time at TUM in the Sommersemester 2018. This course was aimed at advanced Master students, as well as Ph.D. students.

The idea of this course is to give a lecture on algebraic surfaces, a classical topic in advanced algebraic geometry, introducing this interesting classical and modern subject. At the same time, it was also aimed at my graduate studens, which is why the lecture digressed regularly on open questions and research projects. Therefore, it starts with revising the theory of algebraic curves and their classification. From there, it went on to the theory of algebraic surfaces, culminating in the Enriques-Kodaira classification. From there, special topics in the theory of algebraic surfaces, some already on research level were discussed. Finally, an outlook to the minimal model program, which aims at a classification of higher dimensional varieties, was given.

Possible variations of this lecture in the future

Since it leads into active fields of research, one has to update such a lecture regularly and accordingly.

Modulbezeichnung	Algebraische Flächen / Algebraic Surfaces
Modulniveau	Master
Semesterdauer	einsemestrig
Angebot	unregelmäßig
Sprache	englisch
Arbeitsaufwand	270 Gesamtstunden 90 Präsenzstunden 180 Eigenstudiumstunden
Beschreibung der Prüfungsleistung	The examination will consist in a written exam of 60-90 minutes or an oral exam. In both cases, the student will be required to show knowledge of the subject by stating definitions and proving results that have been discussed during the lectures. Moreover, part of the exam will be dedicated to solving exercises by applying the main techniques of the course. Since algebraic surfaces are very explicit objects, it is expected that the student has mastered both the theoretical and the practical aspects of this subject.
Voraussetzungen	MA2101 Algebra MA5120 Algebra 2 (Commutative Algebra) MA5107 (Algebraic Geometry)
Angestrebte Lernergebnisse	After successful completion of the module, the students should know the classification of algebraic surfaces, the birational geometry of surfaces (minimal models, blow-ups, Castelnuovo's contraction theorem), and some important classes, such as rational, elliptic, del Pezzo, K3, and general type surfaces. They should be able to know the abstract theory, but also to give explicit examples (hypersurfaces, branched covers), and to compute the basic invariants (Betti numbers, pluri-genera) in explitly given situations.
Inhalt	algebraic curves and algebraic surfaces, the classification of algebraic curves by genus, the theorems of Riemann-Roch for curves and surfaces, the theorems of Noether and adjunction for surfaces, birational geometry of surfaces: blow-ups, Castelnuovo's theorem, minimal models, the Kodaira-Enriques classification of surfaces, special classes of surfaces: rational, elliptic, del Pezzo, K3, Enriques, general type

Lehr- und Lernmethoden	The course is offered by means of classroom-taught lessons, where the lecturer will present definitions and results on the theory of algebraic surfaces. These will be accompanied by a variety of examples that are aimed at giving to the students a better grasp of the subject. There will be also sample classes/exercise sessions, where the students will learn how to apply the techniques seen in class to explicit examples. Also, the exercise classes will expand on the material of the lectures by providing applications of the main theory to similar settings. An exercise sheet will be provided about a week prior to the corresponding exercise class, and solutions to the full set of exercises will be then provided in class.
Medienform	Blackboard, assignments
Literatur	Badescu, *Algebraic Surfaces*, Beauville, *Complex Algebraic Surfaces*, Hartshorne, *Algebraic Geometry*
Modulverantwortlicher	Christian Liedtke
Lehrveranstaltungen	Vorlesung 4 SWS Übungen 2 SWS

3.5 Certificates

I took the following seminars and courses mentioned in Section 2.4.

Tenure Track Module T2
Implementing Concepts of Teaching and Learning
February 29 - March 1, 2016

Seminar on management
Führend Wissen schaffen - Führungsstil und Führungskompetenz
September 18, 2015

Tenure Track Module T1
Concepts of Teaching
June 29 - 30, 2015

Seminar on argumentation techniques
Kunstgriffe der Argumentation - Schlagfertig im Lehr- und Wissenschaftsbetrieb
January 14, 2015

Tenure Track Module R1
Safeguarding the rules of scientific practice and enhancing your visibility as a researcher
December 10, 2014

Professional teaching counseling
Professionelle Lehrberatung
November 25, 2013

An earlier version of this teaching portfolio contained the certificates. I have removed them from this document but they are still available upon request.