Ex 1)

Elements in $\tilde{M}(D_+(x_i))$ are degree-0-elements of the form $\frac{f}{x_i^d}$ ($f \in M_d$, $d \geq 0$).
Then $\frac{f}{x_i^d} = \frac{1}{x_i^{i_0}}\left(x_i^{i_0 - d} f\right) = 0$ as $x_i^{i_0-d} f = 0$ in M_{i_0}.
$\Rightarrow \tilde{M}(D_+(x_i)) = 0$.
$\Rightarrow \tilde{M} = 0$ as this holds over some open cover.

Ex 2)

(1) Claim:

Let M be a coherent $\mathcal{O}_X$-module on the noeth. scheme X. Then the set
$\{x \in X \mid M_x = 0\} \subseteq X$ is open.

Proof:

This can be checked locally. Thus assume $X = \operatorname{Spec} R$ affine and $M = \tilde{M}$ for some R-module M. Choose finitely many generating elements $\{f_i\} \in M$.
Then $\{x \in X \mid (f_i)_x = 0\} \subseteq X$ is open by ex. 1 on sheet 2.
$\Rightarrow \{x \in X \mid \tilde{M}_x = 0\} = \bigcap_i \{x \in X \mid (f_i)_x = 0\}$ is open in X.

Now assume $\mathcal{F}_x$ is a free $\mathcal{O}_{X,x}$-module. Choose a basis $\{f_i\} \subseteq \mathcal{F}_x$.
Then there is an open subscheme $U \subseteq X$ s.th. all f_i admit representatives over U. Now consider the morphism

$$\varphi : \bigoplus_i \mathcal{O}_U \cdot f_i \longrightarrow \mathcal{F}|_U$$

Then $\ker\varphi$ and $\operatorname{coker}\varphi$ are coherent $\mathcal{O}_U$-modules with
$(\ker\varphi)_x = 0 = (\operatorname{coker}\varphi)_x$.
Hence there are open subsets $V_1, V_2 \subseteq U$ (by our claim) with $x \in V_1, V_2$ s.th.
$\ker\varphi|_{V_1} = 0$ and $\operatorname{coker}\varphi|_{V_2} = 0$
$\Rightarrow \varphi|_{V_1 \cap V_2}$ is an isomorphism!
$\Rightarrow \mathcal{F}$ is free on $V_1 \cap V_2$.

(2) "$\Leftarrow$" was just proved in (1).
For "$\Rightarrow$" choose an open cover $\{U_i\}$ of X s.th. $\mathcal{F}|_{U_i}$ is free $\forall i$. Let $x \in X$ and choose U_i open nbhd of x. Choose an isomorphism $\mathcal{F}|_{U_i} \overset{\varphi}{\cong} \mathcal{O}_{U_i}^d$. Then

$\varphi_x : \mathcal{F}_x = \mathcal{F}(U_i)_x \cong (\mathcal{O}_{U_i}^d)_x = (\mathcal{O}_{X,x})^d$ is again an isomorphism.

Ex 3)

We check that $\Omega_{S/R} \otimes_S (S \otimes_R R')$ has the universal property for $\Omega_{S\otimes_R R'/R'}$:
Let M be any $S\otimes_R R'$-module and $\partial : S\otimes_R R' \to M$ any R'-derivation. Then

$$S \xrightarrow{s \mapsto s\otimes 1} S\otimes_R R' \xrightarrow{\partial} M$$

is a R-derivation (when we view M as a S-module). In particular we get a commutative diagram

$$\begin{tikzcd} S \arrow[r, "d_S"] \arrow[dr, "\partial"'] & \Omega_{S/R} \arrow[d, "f"] \\ & M \end{tikzcd}$$

for some uniquely determined $f : \Omega_{S/R} \to M$ (which is S-linear).
Via $\mathrm{Hom}_S(\Omega_{S/R}, M) \cong \mathrm{Hom}_{S\otimes_R R'}(\Omega_{S/R} \otimes_S (S\otimes_R R'), M)$ (scalar-restriction & scalar extension adjunction) f induces a unique commutative diagram of $S\otimes_R R'$-modules

$$\begin{tikzcd} S\otimes_R R' = S\otimes_S (S\otimes_R R') \arrow[r, "d_S \otimes \mathrm{id}_{S\otimes R'}"] \arrow[dr, "\partial"'] & \Omega_{S/R} \otimes_S (S\otimes_R R') \arrow[d, "f\otimes \mathrm{id}_{S\otimes R'}"] \\ & M \end{tikzcd}$$

$\Rightarrow$ $\Omega_{S/R} \otimes_S (S\otimes_R R')$ has the universal property of $\Omega_{S\otimes_R R'/R'}$.

Ex 4)

Define: $\tilde{d}^i : \prod_{j=1,\dots,i} \Omega_{S/R} \longrightarrow \Lambda^{i+1}\Omega_{S/R}$ (with $\lambda_j \in S$, $b_j \in S$).
$(\lambda_j\, db_j) \longmapsto d(\lambda_1 \cdot \ldots \cdot \lambda_i) \wedge db_1 \wedge \ldots \wedge db_i$. (+ extend additively)

Check: $\tilde{d}^i(\lambda_1 d(b_1 + b_1'), \lambda_2 db_2, \ldots) = \tilde{d}^i(\lambda_1 db_1, \lambda_2 db_2, \ldots) + \tilde{d}^i(\lambda_1 db_1', \lambda_2 db_2, \ldots)$

$\tilde{d}^i(\lambda_1 d(b_1 \cdot b_1'), \lambda_2 db_2, \ldots) = \tilde{d}^i((\lambda_1 b_1) db_1', \lambda_2 db_2, \ldots) + \tilde{d}^i((\lambda_1 b_1') db_1, \lambda_2 db_2, \ldots)$

$\Rightarrow \tilde{d}^i$ is well-defined (as these properties hold for each component)

Check: $\tilde{d}^i(\lambda\lambda_1 db_1, \lambda_2 db_2, \lambda_3 db_3, \ldots) = \tilde{d}^i(\lambda_1 db_1, \lambda\lambda_2 db_2, \lambda_3 db_3)$ and $\tilde{d}^i$ alternating

$\Rightarrow \tilde{d}^i$ factors over $d^i : \Lambda^i \Omega_{S/R} \to \Omega^{i+1}\Omega_{S/R}$ (as these properties hold for each component)

$d^{i+1} \circ d^i = 0$ follows directly from $d1 = 0$.