Some more problems discussed during exercise classes

P9: Assume $\operatorname{char} K = p > 0$.

For positive integers $r, s \geq 1$ we define a group G with underlying variety K^2 and group structure given by

$$(x, y) \cdot (x', y') = (x + x', y + y' + x^{(p^r)} \cdot x'^{(p^s)})$$

Show that this is a unipotent group and embed it into some $U_n \subseteq GL_n$. For $r \neq s$ this group is not abelian.

Remark: In $\operatorname{char} K = 0$ any 2-dim. unipotent group is automatically abelian and isomorphic to $\mathbb{G}_a^{\times 2}$. The example above shows, that the situation is far ~~less~~ more complicated in positive characteristic.

P10:

Set $T = D_n \cap SL_n \subseteq SL_n$.

a) Compute $X(T)$.

b) Describe the morphism

$$X(D_n) \longrightarrow X(T)$$

$$(\chi : D_n \to \mathbb{G}_m) \longmapsto (T \hookrightarrow D_n \xrightarrow{\chi} \mathbb{G}_m).$$

(image? kernel? ,...)

P11:

Let G be an algebraic group with $G = (G, G)$. Show that $X(G) = \operatorname{Hom}(G, \mathbb{G}_m) = \{1\}$ is the trivial group.

P12:

a) List all conjugacy classes in SL_2

b) Compute their dimensions.

c) Which conjugacy classes are closed? Which open? What are the closures of the conjugacy classes?

d) Describe the variety underlying each conjugacy class (i.e. get enough knowledge about them to ~~have a pi~~ be able to picture them in your mind / draw them).

e*) Guess which conjugacy classes define affine algebraic varieties and which do not.