Technische Universität München
Zentrum Mathematik

WS 2014/15
Sheet 14

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Linear Algebraic Groups (MA 5113)

Exercises (just for your entertainment and not to be turned in)

In all exercises K denotes an algebraically closed field and all algebraic groups are defined over this field.

E 53 **(Radicals and normal subgroups)** Let G be an algebraic group and $N \subset G$ a closed normal subgroup.

(a) Show that the image of $R(G)$ under the canonical projection $G \to G/N$ lies inside $R(G/N)$. Prove the same statement for unipotent radicals.

(b) Prove that $R(N) = (R(G) \cap N)^\circ$ and $R_u(N) = (R_u(G) \cap N)^\circ$.

E 54 **(An example)** Fix two integers $m, n \geq 1$. Compute $R(G)$ and $R_u(G)$ for the algebraic group

$$G = \begin{pmatrix} SL_m & K^{m \times n} \\ 0 & GL_n \end{pmatrix} \subset GL_{m+n}.$$

E 55 **(Simple faithful representations)** Let G be a connected algebraic group and V a finite-dimensional faithful representation of G. Assume that V is simple, i.e. that there is no proper non-trivial subspace $W \subset V$ stable under G. Show that G is reductive.
Deduce that the group SO_n is reductive if char $K \neq 2$.

E 56 **(Root systems)** Work out the details of the computation of the root systems for GL_n and SL_n.

Solutions to the exercises will be available from January 29, 2015 on, at

https://www-m11.ma.tum.de/lehre/wintersemester-201415/ws1415-linear-algebraic-groups/