Technische Universität München
Zentrum Mathematik

WS 2014/15
Sheet 5

Prof. Dr. Gregor Kemper
M. Sc. Stephan Neupert

Linear Algebraic Groups (MA 5113)

Exercises (to be turned in: Wednesday, 12.11.2014, during the lecture)

In all exercises K denotes an algebraically closed field and all algebraic groups are defined over this field.

E 17 (Examples of quotients)

(a) Let $T_n \subset GL_n$ be the subgroup of upper triangular matrices and $U_n \subset GL_n$ the subgroup of all upper triangular matrices with all diagonal elements equal to 1. Show that the quotient T_n/U_n is an affine variety and describe this variety explicitly.

(b) Let $T \subset SL_2$ be the subgroup of all upper triangular matrices with determinant 1. Show then that the quotient SL_2/T is isomorphic to the projective line $\mathbb{P}^1$.

E 18 (Irreducibility of quotients) Let G be an algebraic group and $H \subset G$ a closed subgroup. Show that the following statements are equivalent:

(i) The quotient G/H is irreducible.

(ii) For all connected components $Z \subset G$, the intersection $Z \cap H$ is non-empty.

(iii) G is generated by H and the identity component G^0.

E 19 (Computing the Jordan decomposition) Compute the (multiplicative) Jordan decomposition of the element

$$g = \begin{pmatrix} -1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & -2 & -1 \end{pmatrix} \in SL_3$$

Hint: Start by computing the additive Jordan decomposition of g.

E 20 (Jordan decomposition of SL_2) Consider the algebraic group SL_2 and the set of all unipotent elements SL_{2u} respectively all semi-simple elements SL_{2s}.

(a) Show that neither SL_{2u} nor SL_{2s} is a subgroup.

(b) Show that SL_{2u} is not open in SL_2.

(c) Show that SL_{2s} is not closed in SL_2.

(d) Show that SL_{2s} is not open in SL_2.

Solutions to the exercises will be available from November 13, 2014 on, at

https://www-m11.ma.tum.de/lehre/wintersemester-201415/ws1415-linear-algebraic-groups/

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