Technische Universität München
Zentrum Mathematik

WS 2014/15
Sheet 8

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Linear Algebraic Groups (MA 5113)

Exercises (to be turned in: Wednesday, 3.12.2014, during the lecture)

In all exercises K denotes an algebraically closed field and all algebraic groups are defined over this field.

E 29 **(Counterexample to Lie-Kolchin)** Give an example of a solvable algebraic group G and a representation $\rho : G \to GL(V)$ such that V has no common eigenvector for all elements of G.

E 30 **(Solvable and unipotent groups)** Let G be a connected algebraic group. Show that G is solvable if and only if $\mathcal{D}(G) = (G,G)$ is unipotent.

E 31 **(Normal series)** Let G be a connected solvable algebraic group. Show that there is a sequence

$$H_n = \{e\} \subset \ldots \subset H_2 \subset H_1 \subset H_0 = G$$

of normal subgroups $H_i \subset G$, such that for each i the quotient H_i/H_{i+1} are either isomorphic to $\mathbb{G}_m$ or to $\mathbb{G}_a$.

E 32 **(1-dimensional groups)** Let G be a connected algebraic group of dimension 1.

(a) Show that G is either abelian or has only finitely many conjugacy classes.

(b) Show that G is either abelian or unipotent.
Hint: In the second case, show that the characteristic polynomial is constant on G, once one views G as a subgroup of some GL_n.

(c) Show that G is either isomorphic to $\mathbb{G}_m$ or to $\mathbb{G}_a$.
Hint: Use that G is solvable.

Solutions to the exercises will be available from December 4, 2014 on, at

`https://www-m11.ma.tum.de/lehre/wintersemester-201415/ws1415-linear-algebraic-groups/`
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