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Topology (MA 3241)

Exercises (to be handed in Thursday, 14.1.2016, before the lecture)

A 30 (a) Compute the pushout of groups $\mathbb{Z} *_{\mathbb{Z}\times\mathbb{Z}} \mathbb{Z}$ for the morphisms $f_1 : \mathbb{Z}\times\mathbb{Z} \to \mathbb{Z}$, $(x,y) \to 2x+3y$ and $f_2 : \mathbb{Z}\times\mathbb{Z} \to \mathbb{Z}$, $(x,y) \to 3x+2y$.

(b) Let $f : G_0 \to G_1$ be any morphism of groups and denote the trivial group by $\{e\}$. Let H be the normal closure of $f(G_0)$ in G_1, i.e. the smallest normal subgroup of G_1 containing $f(G_0)$. Show that

$$G_1 *_{G_0} \{e\} \cong G_1/H$$

A 31 Solve exercise A29b) now, i.e. compute the fundamental group of the Klein bottle $K = I \times I/\sim$, where $(x,0) \sim (x,1)$ and $(0,y) \sim (1,1-y)$, using the theorem of Seifert-van Kampen.

A 32 Compute the fundamental group of the following spaces in any way you want.

(i) The real projective space $\mathbb{R}P^n = \mathbb{S}^n/(x \sim -x)$ for $n \geq 2$.

(ii) The union of three circles touching each other, i.e. $X = \partial B_{\frac{\sqrt{3}}{2}}(1) \cup \partial B_{\frac{\sqrt{3}}{2}}(\zeta_3) \cup \partial B_{\frac{\sqrt{3}}{2}}(\zeta_3^2) \subset \mathbb{C}$, where ζ_3 is a primitive third root of unity.

(iii) The Möbius strip $M = I \times I/\sim$, where $(0,y) \sim (1,1-y)$ for all $x \in I$.

Solutions to the exercises will be available from January 14, 2016 on, at

`https://www-m11.ma.tum.de/viehmann/topology/`