Technische Universität München
Zentrum Mathematik

WS 2015/16
Sheet 12

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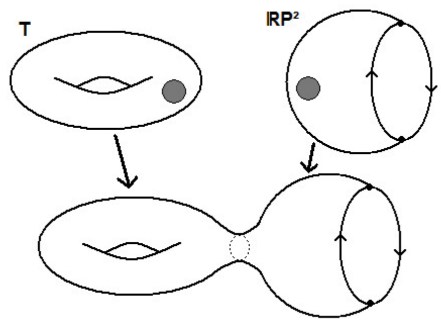
Topology (MA 3241)

Exercises (to be handed in Thursday, 21.1.2016, before the lecture)

A 33 Let X be the connected sum of a torus T and $\mathbb{R}P^2$.



(a) Compute its fundamental group $\pi_1(X)$.

(b) Show that the abelianization $\pi_1(X)^{ab}$ is isomorphic to $\mathbb{Z} \times \mathbb{Z} \times (\mathbb{Z}/2\mathbb{Z})$.

(c) Prove that X is neither homeomorphic to S_g for any $g \geq 1$ nor to N_k for any $k \neq 3$.

A 34 Let $f : X \to B$ and $g : B' \to B$ be morphisms of topological spaces. Let $f' : X' = X \times_B B' \to B'$ be the pullback of f along g.

(a) Assume that f is a fiber bundle with typical fiber F. Prove that f' is again a fiber bundle with typical fiber F.

(b) Assume that g is a covering morphism between path-connected and locally path-connected Hausdorff spaces and that f' is a fiber bundle with typical fiber F. Prove that f is a fiber bundle with the same typical fiber.

A 35 Consider the Grassmannian $Gr_k(\mathbb{C}^n)$ and the space

$$X = \{(V, v) \in Gr_k(\mathbb{C}^n) \times \mathbb{C}^n \mid v \in V\}$$

together with the projection on the first factor $pr : X \to Gr_k(\mathbb{C}^n)$. Prove that X is a fiber bundle over $Gr_k(\mathbb{C}^n)$ with typical fiber $\mathbb{C}^k$.

Hint: For any k-dimensional space $W \subset \mathbb{C}^n$ with orthogonal complement $W^\perp$ consider the subspace

$$U_W = \{V \in Gr_k(\mathbb{C}^n) \mid V \cap W^\perp = 0\} \subset Gr_k(\mathbb{C}^n).$$

Show first that U_W is open and that $pr : X \to Gr_k(\mathbb{C}^n)$ trivializes over U_W.

Solutions to the exercises will be available from January 21, 2016 on, at

https://www-m11.ma.tum.de/viehmann/topology/