Technische Universität München
Zentrum Mathematik

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Sheet 14

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Topology (MA 3241)

Some open-ended Exercises (not to be handed in)

A 38 Consider $X = \mathbb{R}$ together with the topology where a non-empty subset $U \subset X$ is open if and only if there is some $N \gg 0$ such that $(-\infty, N) \cup (N, \infty) \subset U$.
Check that this indeed defines a topology on X. What properties does X have and which are not satisfied?

A 39 Fix $N \in \{0, 1, 2, 3, 4\}$. Let X, Y, Z be topological spaces satisfying the separation axiom (TN) (for the chosen number N) and let $f : X \to Z$ and $g : Y \to Z$ two continuous morphisms. Does it follow that the fiber product space $X \times_Z Y$ is again (TN)?
Warning: The case $(T4)$ is difficult!

A 40 Consider a cube $I^3 \subset \mathbb{R}^3$ and identify each pair of opposite faces. What is the fundamental group of this space? What happens if you identify the faces in some other fashion?

A 41 Fix a unit vector $e \in \mathbb{S}^2 \subset \mathbb{R}^3$ and consider the map $SO(3) \to \mathbb{S}^2$ given by mapping $A \in SO(3)$ to the vector $A \cdot e \in \mathbb{S}^2$. Show that this is a fibration. What are the fibers? What more can you say about this morphism?