Technische Universität München
Zentrum Mathematik

WS 2015/16
Sheet 2

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Topology (MA 3241)

Exercises (to be handed in Thursday, 29.10.2015, before the lecture)

In all exercises $\mathbb{R}^n$ denotes the n-dimensional real vector space endowed with the topology induced by the euclidean metric.

A 4 (a) Let $f : X \to Y$ be a map between two topological spaces. Show that f is a homeomorphism if f is bijective, continuous and open.

(b) Let m, n be positive integers. Show that $\mathbb{R}^m \times \mathbb{R}^n$ with the product topology is homeomorphic to $\mathbb{R}^{m+n}$.

A 5 Let $f : X \to Y$ be a map between two topological spaces and define its graph to be

$$\Gamma(f) = \{(x, f(x)) \in X \times Y \mid x \in X\} \subset X \times Y$$

endowed with the subspace topology. Let $pr_1 : X \times Y \to X$ be the projection onto the first factor. Show that f is continuous if and only if the restriction $pr_1|_{\Gamma(f)} : \Gamma(f) \to X$ of pr_1 to the graph of f is a homeomorphism.

A 6 Consider $\mathbb{R}^2$ together with the equivalence relation $(x, 0) \sim (x', 0)$ for all $x, x' \in \mathbb{R}$. Let $X = \mathbb{R}^2/\sim$ and $p \in X$ the image of $(0,0)$ under the canonical projection $pr : \mathbb{R}^2 \to X$.

(a) Let $U = \{(x,y) \in \mathbb{R}^2 \mid |x \cdot y| < 1\} \subset \mathbb{R}^2$ and $V = \{(x,y) \in \mathbb{R}^2 \mid x < y\} \subset \mathbb{R}^2$. Decide whether $pr(U)$ and $pr(V)$ are open neighborhoods of p in X.

(b) Find a neighborhood basis of $p \in X$.

(c) Show that X is not first-countable.

Solutions to the exercises will be available from October 29, 2015 on, at

`https://www-m11.ma.tum.de/viehmann/topology/`