Technische Universität München
Zentrum Mathematik

WS 2015/16
Sheet 3

Prof. Dr. Eva Viehmann
M. Sc. Stephan Neupert

Topology (MA 3241)

Exercises (to be handed in Thursday, 5.11.2015, before the lecture)

A 7 Give examples of topological spaces X_i and subspaces M_i such that:

(a) $M_1 \subset X_1$ is connected, but its boundary ∂M_1 is not connected.

(b) $M_2 \subset X_2$ is connected, but its interior $\mathring{M}_2$ is not connected.

(c) $M_3 \subset X_3$ is not connected, but its boundary ∂M_3 is connected.

(d) $M_4 \subset X_4$ is not connected, but its interior $\mathring{M}_4$ is connected.

(e) $M_5 \subset X_5$ and $M_5' \subset X_5$ are connected, but their intersection $M_5 \cap M_5'$ is not connected.

A 8 Let $f : X \to Y$ be a quotient map of topological spaces, i.e. $Y = X/\sim$ for some equivalence relation $\sim$. Assume that Y is connected and that for every point $y \in Y$ the fiber $f^{-1}(y) \subset X$ (endowed with the subspace topology) is connected.
Show that X itself is connected as well.

A 9 Prove that there exists no homeomorphism $f : \mathbb{R} \to \mathbb{R}^n$ for $n \geq 2$ (for the euclidean topology on both spaces).
Hint: Remove one point from both spaces and then compare them.

Solutions to the exercises will be available from November 5, 2015 on, at

https://www-m11.ma.tum.de/viehmann/topology/