Technische Universität München
Zentrum Mathematik

WS 2015/16
Sheet 4

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Topology (MA 3241)

<u>Exercises</u> (to be handed in Thursday, 12.11.2015, before the lecture)

A 10 Consider the following two topological spaces X and Y:

(i) $X = \mathbb{R} +_{\mathbb{R}\setminus\{0\}} \mathbb{R}$ is the line with a double point. More explicitly, X has the underlying set of points $(\mathbb{R}\setminus\{0\}) \cup \{0_1, 0_2\}$ and the unique topology, such that the two canonical maps $\mathbb{R} \to X$ mapping 0 to either 0_1 or 0_2 are continuous and open.

(ii) Y is the set of real numbers $\mathbb{R}$ with the topology given by the basis

$$\mathcal{B} = \{(a,b) \mid a < b\} \cup \{(a,b) \cap (\mathbb{R}\setminus Z) \mid a < b\}$$

where $Z = \{\frac{1}{n} \mid n \in \mathbb{Z}, n > 0\}$.

Decide for both spaces, which of the separation properties $(T0), (T1), (T2), (T3)$ and $(T4)$ they satisfy.

A 11 (a) Prove that on every set X, there exists a unique coarsest topology $\mathcal{T}$ such that $(X, \mathcal{T})$ is $(T1)$.

(b) Show that the topological space $(X, \mathcal{T})$ defined in (a) is compact.

(c) Let Y be a topological space, which is $(T2)$. Prove that every sequence in Y converges to at most one point in Y.

A 12 Let X be any topological space, Y be compact and $f : X \to Y$ a map.

(a) Prove that the projection $pr_1 : X \times Y \to X$ is a closed map.

(b) Prove that $f : X \to Y$ is continuous if its graph $\Gamma(f)$ (cf. exercise A5) is closed in $X \times Y$.

(c) Give counterexamples to the assertions in (a) and (b), if Y is allowed to be non-compact.

Solutions to the exercises will be available from November 12, 2015 on, at

`https://www-m11.ma.tum.de/viehmann/topology/`