Technische Universität München
Zentrum Mathematik

WS 2015/16
Sheet 7

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M. Sc. Stephan Neupert

Topology (MA 3241)

There will be no lecture Thursday, 3.12.2015! So solutions cannot be handed in there, but have to be given directly to Stephan Neupert, e.g. by bringing them to his office 02.10.037. The exercise sheet 8 will be available on the homepage from 3.12.2015 on.

Exercises (to be handed in Thursday, 3.12.2015, directly to Stephan Neupert)

A 18 In $GL_n(\mathbb{C})$ consider the subgroups

$$P_k(\mathbb{C}) = \{A = (a_{ij}) \in GL_n(\mathbb{C}) \,|\, a_{ij} = 0 \;\forall\, i > k, j \leq k\} \qquad \text{for } 1 \leq k \leq n-1$$
$$B(\mathbb{C}) = \{A = (a_{ij}) \in GL_n(\mathbb{C}) \,|\, a_{ij} = 0 \;\forall\, i > j\}$$

(a) Show that for all k there exists a homeomorphism $Gr_k(\mathbb{C}^n) \cong GL_n(\mathbb{C})/P_k(\mathbb{C})$.

(b) Show that the flag space

$$\mathcal{F}l(\mathbb{C}^n) = \{(V_i)_{i=1,\dots,n} \,|\, \forall i \; V_i \subset \mathbb{C}^n \text{ subvector space with } \dim V_i = i\,,\; V_i \subset V_{i+1}\}$$

can (as a set) be identified with $GL_n(\mathbb{C})/B(\mathbb{C})$. Use this (or any other way) to prove that $GL_n(\mathbb{C})/B(\mathbb{C})$ is compact.

A 19 Let G be a topological group and $H \subset G$ a subgroup.

(a) Prove that the closure $\overline{H} \subset G$ is a subgroup as well.

(b) Assume $H \subset G$ is normal. Prove that $\overline{H} \subset G$ is normal, too.

(c) Assume G is Hausdorff and H is abelian. Prove that $\overline{H} \subset G$ is abelian, too.

A 20 (a) Show that if a topological group G is $(T0)$, then it is $(T2)$.

(b) Let G be any topological group and $U \subset G$ an open neighborhood of the identity element $e \in G$. Let $U^{-1} \cdot U = \{g^{-1} \cdot h \,|\, g, h \in U\}$ or equivalently $U^{-1} \cdot U = m(i(U) \times U)$ for the inverse map i and the multiplication map m. Prove that $\overline{U} \subset U^{-1} \cdot U$.

(c) Show that every topological group G satisfies $(T3)$.

Solutions to the exercises will be available from December 3, 2015 on, at

https://www-m11.ma.tum.de/viehmann/topology/