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Topology (MA 3241)

Exercises (to be handed in Thursday, 10.12.2015, before the lecture)

A 21 Let $f_1, f_2 : X \to X'$ and $g_1, g_2 : Y \to Y'$ be morphisms such that $f_1 \simeq f_2$ and $g_1 \simeq g_2$ are homotopic.

(a) Prove that $f_1 + g_1 \simeq f_2 + g_2 : X + Y \to X' + Y'$ are homotopic.

(b) Prove that $f_1 \times g_1 \simeq f_2 \times g_2 : X \times Y \to X' \times Y'$ are homotopic.

(c) Assume that all spaces X, X', Y and Y' are locally compact. Prove that

$$g_1 \circ (-) \circ f_1 \simeq g_2 \circ (-) \circ f_2 : \mathrm{Hom}(X', Y) \to \mathrm{Hom}(X, Y')$$

are homotopic, when the homomorphism spaces are endowed with the CO-topology.

A 22 (a) Show that $\mathrm{GL}_2(\mathbb{R})^+ = \{g \in \mathrm{GL}_2(\mathbb{R}) \,|\, \det(g) > 0\}$ and $\mathrm{SL}_2(\mathbb{R})$ are homotopy equivalent.

(b) Show that $\mathrm{GL}_2(\mathbb{R})^+$ and $\mathrm{SO}_2(\mathbb{R})$ are homotopy equivalent.

(c) Conclude that $\mathrm{GL}_2(\mathbb{R})$ is homotopy equivalent to $\mathbb{S}^1 + \mathbb{S}^1$.

Solutions to the exercises will be available from December 10, 2015 on, at

https://www-m11.ma.tum.de/viehmann/topology/