Technische Universität München
Zentrum Mathematik

WS 2015/16
Sheet 9

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Topology (MA 3241)

Exercises (to be handed in Thursday, 17.12.2015, before the lecture)

A 24 Let X and Y be topological spaces with base points $x_0 \in X$ and $y_0 \in Y$. Take (x_0, y_0) as a base point of $X \times Y$. Prove that there is a canonical isomorphism of groups

$$\pi_1(X \times Y) \cong \pi_1(X) \times \pi_1(Y).$$

A 25 Let G be a topological group with multiplication $\cdot$ and base point e. Define on $\Omega = \mathrm{Hom}([0,1], \{0,1\}; G, \{e\})$ a multiplication $\diamond : \Omega \times \Omega \to \Omega$ via

$$(\gamma \diamond \gamma')(t) = \gamma(t) \cdot \gamma'(t) \qquad \forall \gamma, \gamma' \in \Omega, t \in [0,1]$$

(a) Prove that $\diamond$ induces a well-defined multiplication on $\pi_1(G)$.

(b) Prove that $\diamond$ coincides with the usual group multiplication $*$ on $\pi_1(G)$.
Hint: Compute $(\gamma c_e) \diamond (c_e \gamma')$, where $c_e \in \Omega$ is the constant function with image e.

(c) Show that $\pi_1(G)$ is commutative.

A 26 Prove or disprove that the following morphisms are coverings:

(i) $f : [0,3] \to [0,1]$, $f(t) = \begin{cases} t & \text{if } t \in [0,1] \\ 2-t & \text{if } t \in (1,2) \\ t-2 & \text{if } t \in [2,3] \end{cases}$

(ii) $q : \mathbb{S}^1 \times I \to M := (\mathbb{S}^1 \times I)/\sim$ the quotient morphism, where $(x,t) \sim (-x, 1-t)$ for all $x \in \mathbb{S}^1$, $t \in I$.
Remark: q maps the cylinder $\mathbb{S}^1 \times I$ to the Möbius strip M.

(iii) $exp : (0, \infty) \to \mathbb{S}^1$, $exp(t) = e^{2\pi i t}$

Solutions to the exercises will be available from December 17, 2015 on, at

https://www-m11.ma.tum.de/viehmann/topology/