Algebraic Geometry

To be handed in January 9, before the lecture.

Exercise 1. Let k be a field.

1. Show that every morphism of schemes $X \to \mathrm{Spec}\, k$ is closed.

2. Find an example where the above morphism is not universally closed. (Hint: one may consider $\mathbb{A}^1_k$ as an example.)

Exercise 2. Describe the fibers of the following morphisms at each point of the base scheme, and determine whether the fiber is irreducible or reduced.

1. Let k be a field. Consider the morphism $\mathrm{Spec}\, k[x,y]/(x^2+y^2) \to \mathrm{Spec}\, k[x]$ induced by the natural ring homomorphism $k[x] \to k[x,y]/(x^2+y^2)$.

2. Consider the structure morphism $\mathrm{Spec}\, \mathbb{Z}[x]/(x^2+1) \to \mathrm{Spec}\, \mathbb{Z}$.

Exercise 3. Verify that being an immersion is a property local on the target, stable under composition and under base change.

Exercise 4. (This exercise is supposed help you to follow the lecture more easily, but also in the preparation for the final exam. It need not be handed in.)

1. Collect all main notions that have been introduced in the lecture: What is a scheme, and what did we need to be able to define schemes? Which properties of schemes did we study? Which main properties of morphisms of schemes did we study?

2. Find examples of schemes/morphisms of schemes having the various properties and also give counterexamples.

3. Which definitions, properties and notions considered in the lecture differed from the intuitive expectation, and why were definitions made in this way? What were typical examples?

Merry christmas and a happy new year 2019!