Technische Universität München
Zentrum Mathematik

WS 2018/19
Sheet 11

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Algebraic Geometry

To be handed in January 16, before the lecture.

Exercise 1. Assume that all occurring schemes are Noetherian. Prove that

1. Open and closed immersions are separated.

2. To be separated is stable under composition.

3. To be separated is local on the target.

Exercise 2. Prove that a morphism of schemes $f : X \to Y$ is quasi-compact if and only if for every affine open subscheme $U \subseteq Y$ the subscheme $f^{-1}(U) \subseteq X$ is quasi-compact.

Exercise 3. Let $f : X \to Y$ and $g : Y \to Z$ be morphisms of Noetherian schemes such that $g \circ f$ is a closed immersion and g is separated. Prove that f is a closed immersion.
(Hint: Factor the morphism over its graph $\Gamma_f : X \to X \times_Z Y$.)

Exercise 4.

1. Let $f : Y \to S$ be separated and let $h, h' : X \to Y$ be two morphisms of schemes with $f \circ h = f \circ h'$. Assume that X is reduced and that there is an open dense subset $U \subseteq X$ with $h|_U = h'|_U$. Prove that $h = h'$. (Hint: Consider the morphism $(h, h') : X \to Y \times_S Y$.)

2. Show that the condition that X is reduced is indeed necessary by giving a counterexample.

In case of questions please send us an email or contact us before or after the seminar/problem session.
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