Technische Universität München
Zentrum Mathematik

WS 2018/19
Sheet 12

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Algebraic Geometry

To be handed in January 23, before the lecture.

Exercise 1. Let k be a field and $X = V(T_2^2 - T_1^3 - T_1) \subseteq \mathbb{A}_k^2 = \mathrm{Spec} k[T_1, T_2]$. Is $X \to \mathrm{Spec} k$ separated, of finite type and proper? Try to find one proof using the definition and one using the valuative criterion for properness.

Exercise 2. Let k be an algebraically closed field and X a proper integral k-scheme.

1. Show that $\mathcal{O}_X(X) = k$. (Hint: Construct morphisms $X \to \mathbb{A}_k^1 \to \mathbb{P}_k^1$ out of sections.)
2. Let $f : X \to Y = \mathrm{Spec} R$ be a map to an affine scheme. Show that $f(X)$ consists of one point.
3. Let R be a finitely generated algebra over a field k, and assume that R is not zero-dimensional. Show that $\mathrm{Spec} R \to \mathrm{Spec} k$ is not proper.

Exercise 3. Compute the tangent spaces $T_x X$ for

1. $X = \mathrm{Spec} k[s,t]/(t^2 + s^3 - s^2)$ and $x = (0,0)$ resp. $x = (1,0)$.
2. $X = \mathrm{Spec} k[[s,t]]/(st, t^2)$ and $x = (0,0)$.

Exercise 4. Let S be a scheme and $X \to S$ be a proper morphism of schemes. Let $Y \to S$ be separated and $f : X \to Y$ an S-morphism. Show that $f(X) \subset Y$ is closed. (Hint: Consider the graph of f.)

In case of questions please send us an email or contact us before or after the seminar/problem session.
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