Technische Universität München
Zentrum Mathematik

WS 2018/19
Sheet 14

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Algebraic Geometry

To be handed in Feburary 6, before the lecture.

Exercise 1. Let k be a field of characteristic 0. Compute the regular locus of

$$X = V((y-1)(x^2+y^2-z^3)) \subseteq \mathbb{A}^3_k.$$

Exercise 2. Show that the following are non-smooth maps between regular schemes:

1. $\mathrm{Spec}\mathbb{Z}[x]/x^2+1 \to \mathrm{Spec}\mathbb{Z}$
2. $\mathbb{A}^1 \to \mathbb{A}^2, x \mapsto (x,0)$

Exercise 3. Let k be a field such that $\mathrm{char}(k) \neq 2$. Let $f(x)$ be an element of $k[x]$ which is not a squre. Set $X = \mathrm{Spec}k[x,y]/y^2 - f(x)$.

1. Show that X is geometric integral. Show that X is smooth over k (and in particular normal) outside the finite k-scheme $V(y) \cap X$.
2. Show that X is normal if and only if f is square free.
3. Determine the normalization of X.

Complementary exercise. Let A be a discrete valuation ring, and $\pi \in A$ a uniformizer. (One may take as example $A = \mathbb{Z}_p$ and $\pi = p$.) Let $f(x) \in A[x]$ be an *Eisenstein* polynomial, i.e. $f(x) = x^n + a_{n-1}x^{n-1} + ... + a_1 x + a_0$ such that $a_i \in \pi A$ for all i, and $a_0 \notin \pi^2 A$. Show that $A[x]/f$ is a discrete valuation ring.

In case of questions please send us an email or contact us before or after the seminar/problem session.
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