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Algebraic Geometry

To be handed in December 5, before the course.

Exercise 1. Let k be a field.

1. Let $X = \operatorname{Spec}k[x, y]/(x^2 - y^3)$. Compute the dimension of X and a Noether normalization of X (and prove that it has the required properties).
2. Compute the dimension of $V_+(X_1X_2 - X_3X_4) \subseteq \mathbb{P}_k^3$.

Exercise 2. Let $f : X \rightarrow Y$ be a closed morphism of schemes.

1. If f is surjective, then $\dim X \geq \dim Y$.
2. If f is injective, then $\dim X \leq \dim Y$.

Exercise 3. Let $X = \operatorname{Spec}R$ be an affine scheme of finite type over a field k and let $f \in R$.

1. Prove that $\dim X \geq \dim V(f) \geq \dim X - 1$.
2. Give examples showing that both cases can occur.
3. If f is not a zero divisor show that $\dim V(f) = \dim X - 1$.

Exercise 4. Let X be a Noetherian scheme and let $U \subset X$ be an open dense subscheme. Then $\dim X \geq \dim U$.

1. Show that $U = \operatorname{Spec}k[x, y][1/(1 - xy)] \subseteq X = \mathbb{A}_k^2$ is an example of the above situation where $\dim U = \dim X$.
2. Give an example of such X and U with $\dim U < \dim X$.

In case of questions please send us an email or contact us before or after the seminar/problem session.
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