Technische Universität München
Zentrum Mathematik

WS 2018/19
Sheet 8

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Algebraic Geometry

To be handed in December 12, before the lecture.

Exercise 1. Define a scheme SL_n such that for every ring R

$$\mathrm{SL}_n(R) = \{\text{invertible } n \times n\text{-matrices with entries in } R \text{ and determinant } 1\}.$$

Here a matrix is invertible if it has a (two-sided) inverse in the same set of $n \times n$-matrices with entries in R.

Exercise 2. Let X be a scheme and $X = \bigcup_i U_i$ an open covering.

1. Show that for every local ring R we have $X(R) = \bigcup U_i(R)$.
2. Give an example showing that this is in general not true if R is not local.

Exercise 3. Let R be a ring. For every R-algebra A let $\alpha_A : A \to A$ be a map of sets, in such a way that for every ring homomorphism $\varphi : A \to A'$ we have

$$\varphi \circ \alpha_A = \alpha_{A'} \circ \varphi.$$

Show that there is a polynomial $f \in R[T]$ such that $\alpha_A(a) = f(a)$ for every A and every $a \in A$.

Try to find a direct proof, but also one via the Yoneda lemma.

Exercise 4. Let $\mathcal{C}$ be a category in which all fiber products exist. Let S be an object of $\mathcal{C}$ and let X, Y, Z be S-objects let $f : X \to Z$ and $g : Y \to Z$ be S-morphisms. Let $S' \to S$ be a morphism in $\mathcal{C}$ and for every S-object A let $A_{S'} = A \times_S S'$. Prove that there is an isomorphism

$$(X \times_Z Y)_{S'} \cong X_{S'} \times_{Z_{S'}} Y_{S'}.$$

In case of questions please send us an email or contact us before or after the seminar/problem session.
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