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## Introduction to Algebraic Number Theory

### Sheet 1

**Exercise 1.** Find the factorization of  $6 + 12i \in \mathbb{Z}[i]$  into irreducible elements in  $\mathbb{Z}[i]$ .

**Exercise 2.** Let  $a$  be a non-zero element of  $\mathbb{Z}[i]$  and let  $I := (a) := \{ra \mid r \in \mathbb{Z}[i]\}$ . Prove that  $\mathbb{Z}[i]/I$  is finite. If  $a$  is prime show that  $\mathbb{Z}/I$  is an integral domain and deduce that  $I$  is a maximal ideal of  $\mathbb{Z}[i]$ .

**Exercise 3.** Let  $R \subset \mathbb{Q}$  be the subring  $\{m + n\sqrt{-6} \mid m, n \in \mathbb{Z}[i]\}$  and let  $I$  be the ideal of  $R$  generated by 2 and  $\sqrt{-6}$ . Show that  $R/I$  has exactly two elements and deduce that  $I$  is a maximal ideal of  $R$ .