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Introduction to Algebraic Number Theory
Sheet 2

Exercise 1. (a) Show that the following numbers are algebraic:

$$\frac{1}{2}, \sqrt{5}, \sqrt{17} + \sqrt{19}, e^{2\pi i/7}$$

(b) Assuming that the polynomials you have found are irreducible, what are the conjugates of these numbers?

(c) What are the traces and norms of these numbers?

Exercise 2. Let $K = \mathbb{Q}(\alpha)$, where $\alpha^3 = d$ for $d \in \mathbb{Z}$ not a cube. Describe the embeddings σ_1, σ_2, σ_3 of K into $\mathbb{C}$. Are the fields $\sigma_1(K)$, $\sigma_2(K)$ and $\sigma_3(K)$ different?

Exercise 3. Let $f = X^3 + pX + q \in \mathbb{Z}[X]$ be irreducible and α a root of f.

(a) Show that α^2 has minimal polynomial $X^3 + 2pX^2 + p^2 X - q^2$.

(b) Show that $1, \alpha, \alpha^2, \alpha^3$ have traces $3, 0, -2p, -3q$ respectively and compute the trace of α^4.

(c) Show $d(1, \alpha, \alpha^2) = -4p^3 - 27q^2$