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Introduction to Algebraic Number Theory Sheet 4

Exercise 1. Let $K = \mathbb{Q}(\alpha)$, where α is a root of the polynomial $X^3 - X - 4$. Show that

$$\mathcal{O}_K = \mathbb{Z}\left[1, \alpha, \frac{\alpha + \alpha^2}{2}\right].$$

Exercise 2. (a) Show that

$$54 = 2 \cdot 3^3 = \frac{13 + \sqrt{-47}}{2} \cdot \frac{13 - \sqrt{-47}}{2}$$

are two different factorizations of 54 into irreducible factors in $\mathcal{O}_{\mathbb{Q}(\sqrt{-47})}$.

(b) Compute the factorization of (54) into prime ideals in $\mathcal{O}_{\mathbb{Q}(\sqrt{-47})}$.