Technische Universität München
Zentrum Mathematik

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Sheet 5

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Introduction to Algebraic Number Theory

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Exercise 1. Let R be a Dedekind ring. For ideals I and J of R, write $I|J$ for $J \subset I$ and call I and J *coprime* if $I + J = R$.
Let I, J, K be ideals of R such that I and J are coprime

(a) Show that if $I|JK$ then $I|K$.

(b) Show that if $I|K$ and $J|K$ then $IJ|K$.

Exercise 2. Let p be an odd prime number.

(a) Show that -2 is a square in $\mathbb{F}_p$ if and only if $p \equiv 1, 3$ (8).

(b) Show that $\mathbb{Z}[\sqrt{-2}]$ is Euclidean with respect to the function $N_{\mathbb{Q}(\sqrt{-2})/\mathbb{Q}}$.

(c) Show that p can be written as $x^2 + 2y^2$ with $x, y \in \mathbb{Z}$ if and only if $p \equiv 1, 3$ (8).

Hint: For (a), consider a primitive eighth root of unity ζ in an algebraic closure of $\mathbb{F}_p$. Let $y = \zeta - \zeta^{-1}$. Then $y^2 = -2$. Now consider y^p.