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Introduction to Algebraic Number Theory

Sheet 8

Exercise 1. Let $K = \mathbb{Q}(\sqrt[3]{5})$. Given that $\mathcal{O}_K = \mathbb{Z}[\sqrt[3]{5}]$, find the prime factorisation of the ideals (2) , (3) , (5) and (7) in \mathcal{O}_K . Show that all prime ideal factors which occur are principal. Using Minkowski's bound, show that \mathcal{O}_K is a PID.

Exercise 2. Show that the equation $y^2 = x^3 - 5$ has no integer solutions.

Hint: For such x and y , show that the ideals $(y + \sqrt{-5})$ and $(y - \sqrt{-5})$ of $\mathcal{O}_{\mathbb{Q}(\sqrt{-5})}$ are coprime and then use the fact that $\mathbb{Q}(\sqrt{-5})$ has class number two.

Exercise 3. Compute the class number of $\mathbb{Q}(\sqrt{-23})$.