Technische Universität München
Zentrum Mathematik

Prof. Dr. Eva Viehmann
Dr. Paul Ziegler

Introduction to Algebraic Number Theory
Sheet 11

Exercise 1. (a) Let k be a finite field and $|\cdot|\colon k \to \mathbb{R}_{\geq 0}$ an absolute value on k. Show that $|x| = 1$ for all $x \in k$.

(b) Let k be a field of characteristic $p > 0$. Show that there is no archimedean absolute value on k.

Exercise 2. Let K be a field with a non-archimedean absolute value $|\cdot|$. Let $x, y \in K$ with $|x| \neq |y|$. Show that $|x + y| = \max(|x|, |y|)$.

Exercise 3. Let K be a field with a non-archimedean absolute value $|\cdot|$. Let $d(x, y) = |x - y|$ be the associated metric.

(a) For $a \in K$ and $r \in \mathbb{R}_{\geq 0}$ let $D(a, r) = \{x \in K \mid d(x, a) \leq r\}$ be the "closed" disc of radius r around a. Show that $D(a, r)$ is open and closed in K.

(b) Consider two discs $D = D(a, r)$ and $D' = D(a', r')$ in K. Show that either D and D' are disjoint or there exists $b \in K$ such that $D = D(b, r)$ and $D' = D(b, r')$.

(c) Show that every triangle in K is isosceles: For $x, y, z \in K$ with $d(x, z) < d(y, z)$ we have $d(y, z) = d(x, y)$.