Technische Universität München
Zentrum Mathematik

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Sheet 12

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Introduction to Algebraic Number Theory
Sheet 12

Exercise 1. Let R be a discrete valuation ring with valuation v, maximal ideal $\mathfrak{m}$ and quotient field K. For $n \in \mathbb{N}_{>0}$ let $U^{(n)} = 1 + \mathfrak{m}^n = \{x \in K \mid v(1-x) \geq n\}$.

(a) Show that the $U^{(n)}$ are subgroups of R^* which form a neighborhood basis of 1 in R.

(b) Show that there are natural isomorphisms $R^*/U^{(n)} \cong (R/\mathfrak{m}^n)^*$ and $U^{(n)}/U^{(n+1)} \cong R/\mathfrak{m}$.

Exercise 2. Let k be a field and $K = k(t)$ the function field in one variable over k. In analogy with the theorem of Ostrowski, show that up to equivalence, the only valuations of K are the valuations $v_\mathfrak{p}$ associated to a prime ideal $\mathfrak{p} = (p(t))$ of $k[t]$ and the valuation v_∞ sending $f \in K$ to $-\deg(f)$. What are the residue fields of these valuations?