Prof. Dr. Eva Viehmann
Dr. Paul Ziegler

Introduction to Algebraic Number Theory
Sheet 13

Exercise 1. (a) Is there a discretely valued field which is algebraically closed?

(b) Show that $\mathbb{Q}_p \neq \mathbb{Q}$ for every prime p.

Exercise 2. Let K be a complete non-archimedean field, R the valuation ring of K and $\mathfrak{m} \subset R$ the maximal ideal. For a natural number n which is not zero in $R/\mathfrak{m}$ and $u \in R$ satisfying

$$u \equiv 1 \pmod{\mathfrak{m}},$$

show that u is an n-th power in K.

Exercise 3. (a) Show that $a \in \mathbb{Q}_p$ is in $\mathbb{Z}_p^*$ if and only if the polynomial $X^n - a$ has a root in $\mathbb{Q}_p$ for infinitely many $n \geq 0$.

(b) Show that the identity is the only field automorphism of $\mathbb{Q}_p$.