

Talk 1. Overview - Alexander Ivanov

Talk 2. Filtered vector spaces - Bernhard Werner

(Chap. I of [DOR]) The following topics should be covered: basic definitions; semi-stability; semi-stable objects of fixed slope form an abelian subcategory. Tensor product theorem (without proof). Harder-Narasimhan-filtration and μ -polygon.

Talk 3. Period domains over finite fields for GL_n - Robert Mijatovic

Essentially: [DOR], Chap. II§1,3. Also give some examples ([DOR], Chap. II, Example 2.1.7.(by Drinfeld) and 2.1.9).

If time permits, one can also mention related topics: relations to GIT ([DOR] Chap. II §2); the simply-connectedness of period domains ([DOR], Chap. XI, §2; in particular, Proposition 11.2.9); the "non-relation" to Deligne-Lusztig-varieties ([DOR], Chap. XII§2); the elaborate discussion of the Drinfeld example ([DOR], Chap. XII§3).

Talk 4. Filtered isocrystals and period domains for p -adic fields - ???

Basic notions ([DOR] Chap. VIII §1), point out the parallels to the case of a finite field. Fontaine's functor (from crystalline Galois representations to filtered isocrystals) and admissibility; admissible implies weakly admissible (plus the converse without proof).

Recall briefly the rigid-geometric and the Berkovich-analytic setups (here various sources may be used. One possible is [Ha2] Appendix A).

Define the period domains for p -adic fields ([DOR] Chap. VIII §2,3, see also [RZ] Chap. I, 1.35, 1.37), at some points it should be possible to refer to the setting over the finite field, without proving the results once again. It should be explained that these domains do not exist as schemes, but do exist as rigid varieties (construction in [RZ]) and as analytic spaces in the sense of Berkovich (construction in [DOR] Proposition 8.2.1). Compare both constructions ([DOR] Proposition 8.2.4). Explain also the Harder-Narasimhan stratification.

Also, give an example for the above construction.

Define the period morphism for p -divisible groups ([RZ] Chap. V, 5.16; see also [DOR] Chap. XI, §4).

This talk may be take longer than one session.

Talk 5. p -adic Hodge theory I - Christian Liedtke

([Ha2] §3,4). This talk should be an extract from p -adic Hodge theory, explaining Fontaine rings and Kedlaya's theory of ϕ -modules over $\mathbf{B}_{\text{rig}}^+$. The ultimate goal to have in mind should be the application of ϕ -modules in the next talk. Also, one should try to give at least a partial motivation, not only a summary of the technical results quoted [Ha2] §3,4. Therefore use the references cited at the beginning of [Ha2] §3 and [Ked].

Talk 6. p -adic Hodge theory II - ???

([Ha2] §5) This talk is a continuation of the last one. The goal is to associate a ϕ -module $\mathbb{M}(\underline{D})$ over $\tilde{\mathbf{B}}_{\text{rig}}^+$ with a filtered isocrystal $\underline{D}$ (Definition 5.2), to compute its degree (Theorem 5.5) and then to state Theorem 5.9 of Berger (which is taken from [Be]).

Talk 7. Definition of $\check{\mathcal{F}}_b^0$ - Stephan Neupert

([Ha2] §6,7) Define $\check{\mathcal{F}}_b^0$ in the minuscule case and prove that it is a paracompact open analytic subspace of $\check{\mathcal{F}}^{an}$ (Theorem 6.6). Also treat Example 6.7, which shows that the inclusion $\check{\mathcal{F}}_b^0 \hookrightarrow \check{\mathcal{F}}^{an}$ may be strict. State conjectures 6.4 and 6.5 about the structure of $\check{\mathcal{F}}_b^0$.

If time permits, explain why a proper subspace of $\check{\mathcal{F}}^{an}$ is needed. Refer to the above example and [Ha2] §9.

For $G = GL_n$, show that in the minuscule case the period morphism factors through $\check{\mathcal{F}}_b^0$ (Proposition 7.2).

Talk 8. The image of the period morphism I - Paul Hamacher

([SW]) State and prove Theorem 6.2.1 and Proposition 6.2.2. State the necessary results from previous sections, in particular Proposition 5.1.1, Theorem 5.1.4 and Proposition 5.1.6, and sketch their proofs if time permits.

Talk 9. The image of the period morphism II - ???

([Ha2] §7) Then prove the main Theorem 7.3, which in particular shows that the image of the period morphism is precisely $\check{\mathcal{F}}_b^0$ and that the rational Tate-module of the universal p -divisible group over the source of the period morphism descends to a $\mathbb{Q}_p$ -local system on $\check{\mathcal{F}}_b^0$. This also defines the desired tensor functor and tower of étale coverings.

Further topics.

(the following topics can possibly be handled in less than a full session.)

PEL-case. ([Ha2] §8) Introduce the necessary notions (before Theorem 8.4) and prove that the main theorem from talk 9 also holds in the PEL-case.

Weak admissibility vs. admissibility. ([Ha2] §9) For this talk assume $G = GL_n$. Study the conditions on $b \in GL_n(K_0)$, under which $\mathcal{F}_b^0 = \mathcal{F}_b^{wa}$ holds. Theorem 9.4 with an extract of the proof, Corollary 9.4 and Remark 9.5 should be covered.

Connectedness and maximality of $\mathcal{F}_b^0$. ([Ha2] §10) Here the evidence for the conjectures from talks 5 and 8 can be studied.

References

- [Be] Berger L.: *Équations différentielles p -adiques et (ϕ, N) -modules filtrés*, Asterisque **319** (2008), 13-38.
- [DOR] Dat J.-F., Orlik S., Rapoport M.: *Period domains over finite and p -adic fields*, Cambridge Tracts in Mathematics **183**, Cambridge University press, Cambridge, 2010.
- [Fa] Faltings G.: *Coverings of p -adic period domains*, J. Reine Angew. Math. **643** (2010), 111-139.

- [Ha1] Hartl U.: *On period spaces for p -divisible groups*, C. R. Math. Acad. Sci. Paris Ser. I **346** (2008), 1123-1128.
- [Ha2] Hartl U.: *On a conjecture of Rapoport and Zink*, arXiv:math/0605254, 2013.
- [Ked] Kedlaya K.: *Slope filtrations revisited*, Doc. Math. **10** (2005), 447-525.
- [RZ] Rapoport M., Zink Th.: *Period spaces for p -divisible groups*, Ann. Math. Stud. **141**, Princeton Univeristy press, Princeton, 1996.
- [SW] Scholze P., Weinstein J.: *Moduli of p -divisible groups*, arXiv:1211.6357, 2013.