

## Category Theory

### Exercise Sheet 3

Lecture Homepage: <https://www.math.cit.tum.de/algebra/lehre/sommersemester-2022/ss2022-category-theory/>

**Exercise 1.** Find several examples of natural transformations from some of the other lectures which you are attending, and show that these satisfy the definition of a natural transformation.

**Exercise 2.** Let  $F, G: C \rightarrow D$  be two functors and  $\alpha: F \rightarrow G$  be a natural transformation. Show that  $\alpha$  is an isomorphism of functors, i.e. admits a two-sided inverse natural transformation, if and only if for all  $c \in C$ , the morphism  $\alpha_c: Fc \rightarrow Gc$  in  $D$  is an isomorphism.

**Exercise 3.** Let  $G$  and  $H$  be groups. Give concrete descriptions of the following objects:

- Functors  $BG \rightarrow BH$ .
- Functors  $BG \rightarrow \text{Set}$ .
- Functors  $BG \rightarrow C$  for an arbitrary category  $C$ .
- Natural transformations between the above functors.