

Category Theory

Exercise Sheet 5

Lecture Homepage: <https://www.math.cit.tum.de/algebra/lehre/sommersemester-2022/ss2022-category-theory/>

Exercise 1. For some of the examples of universal properties which we have seen in the lecture, find (new) examples of categories in which objects with these universal properties always exist, and also examples of categories in which objects with these universal properties don't always exist.

Exercise 2. Consider the category associated to the preordered set (\mathbb{Z}, \leq) , that is the category whose objects are the elements of \mathbb{Z} and whose morphisms are given by

$$\text{Hom}(n, m) = \begin{cases} \{*\} & \text{if } n \leq m \\ \emptyset & \text{else,} \end{cases}$$

with the only possible composition rule. Describe products and coproducts in this category.

Exercise 3. Is the category of sets equivalent to its opposite category?