

# Workshop “Fluctuating hydrodynamics”

Raitenhaslach 2–4 April 2025

[https://www.math.cit.tum.de/math/forschung/gruppen/analysis-and-mathbio/  
events/fluctuating-hydrodynamics/](https://www.math.cit.tum.de/math/forschung/gruppen/analysis-and-mathbio/events/fluctuating-hydrodynamics/)

## Programme

Day	Wednesday 2.4	Thursday 3.4.	Friday 4.4.
08:30	Shuttle from Hotel Glöcklhofer to Raitenhaslach		
09:00–09:40	Grün	Wu	Martini
09:45–10:25	Popat	Dirr	Zimmer
10:30–11:00	<i>Coffee Break</i>		
11:00–11:40	Pavliotis	Cornalba	Konarovskii
11:45–12:25	Ayala	von Renesse	<i>Lunch</i>
12:30–13:30	<i>Lunch</i>		
13:30–15:30	<i>Discussion &amp; Tea Break</i>		
15:30–16:30	Gerber & Schlichting		
17:15	Shuttle to Hotel		

### Practical information

- The meeting takes place at the Raitenhaslach conference centre.
- Accommodation: Hotel Glöcklhofer Ludwigsberg 4, 84489 Burghausen, phone +49 8677 916 400.
- There is a shuttle service between the hotel and Raitenhaslach, leaving **8:30** at the hotel and returning **17:15** from Raitenhaslach.
- Meals:
  - Lunch will be taken in Raitenhaslach.
  - On Wednesday there will be a buffet dinner at Hotel Glöcklhofer at 19:00.
  - A workshop dinner will be held at the Augustiner Brückenwirt, Bruckgasse 104, Burghausen on Thursday, 19:00 pm.
- The meeting finishes with lunch on Friday.
- Shuttles for Thursday to return from Raitenhaslach will be organised during the meeting.



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## List of Abstracts

### Hydrodynamic Limits and Non-Equilibrium Fluctuations for the Symmetric Inclusion Process with Long Jumps

Mario Ayala

Technische Universität München

In this talk, we focus on establishing scaling limits for the symmetric inclusion process introduced in [2]. Specifically, we aim to derive hydrodynamic limits and investigate non-equilibrium fluctuations in the case where particles can interact with others arbitrarily far apart, allowing for so-called long jumps.

Hydrodynamic limits for the exclusion process with long jumps were first established by [4], who formulated a Cauchy problem associated with the infinitesimal generator of jump type. This contrasts with the case of short jumps, where the Cauchy problem is associated with a generator of diffusive type. Subsequently, [3] investigated fluctuations from the hydrodynamic limit for the exclusion process, considering initializations from Bernoulli product measures. They showed that the scaling limit of density fluctuations corresponds to a fractional generalized Ornstein–Uhlenbeck process.

Despite the fundamental differences in their nature, the inclusion process shares similarities with the exclusion process in the context of scaling limits. Both processes exhibit a similar structure in terms of “short-jump” hydrodynamics and equilibrium fluctuations [1]. In this work, we demonstrate that these structural similarities persist even in the long-jump setting. First, we establish that, under appropriate rescaling, the hydrodynamic equation for the long-jump version coincides with that of the exclusion process, sharing the same underlying random walker. Moreover, our main contribution is the establishment of non-equilibrium fluctuations. We show that the density fluctuation field, starting from a class of appropriate non-equilibrium measures, converges to a time-dependent generalized Ornstein–Uhlenbeck process. Notably, the characteristics of this process can be verified to coincide, modulo a constant, with those found in [3] for the exclusion process with long jumps when simplified to stationarity.

Joint work with Johannes Zimmer.

## References

- [1] Mario Ayala, Gioia Carinci, and Frank Redig. Higher order fluctuation fields and orthogonal duality polynomials. *Electronic Journal of Probability*, 26:1–35, 2021.
- [2] Cristian Giardinà, Jorge Kurchan, and Frank Redig. Duality and exact correlations for a model of heat conduction. *Journal of mathematical physics*, 48(3):033301, 2007.
- [3] Patricia Gonçalves and Milton Jara. Density fluctuations for exclusion processes with long jumps. *Probability Theory and Related Fields*, 170:311–362, 2018.
- [4] Milton Jara. Nonequilibrium scaling limit for a tagged particle in the simple exclusion process with long jumps. *Communications on pure and applied mathematics*, 62(2):198–214, 2009.

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## Numerical aspects for stochastic PDEs of Fluctuating Hydrodynamics

Federico Cornalba  
University of Bath

Interacting particle systems can be often be viewed as (exact) solutions to (singular) stochastic PDEs. Despite these SPDEs being singular, their numerical approximations offer computational efficiency (over of the particle systems) when it comes to estimating fluctuations in relevant scaling regimes. We will detail this in the case of the Dean-Kawasaki model (based on joint works with Julian Fischer, Jonas Ingmanns, Claudia Raithel). We will then present some open problems and aspects of interest (concerning numerics for this class of SPDEs), and we look forward to discussing these with the participants of the workshop.

# Breakdown of the mean-field description of interacting systems: Phase transitions, metastability, and coarsening

Nicolai Gerber & André Schlichting

Universität Ulm

We present results concerning the qualitative and quantitative description of interacting particle systems, with particular emphasis on those possessing a phase transition under the change of relevant system parameters. For this, we first discuss and identify continuous and discontinuous phase transitions for mean-field limits of interacting particle systems on the torus and spheres. Since phase transitions are intimately related to the metastability of the stochastic particle system, we show how a suitable mountain pass theorem in the space of probability measures can describe the metastable behaviour of the underlying finite particle system. We also show that the mean-field description of the particle system in the regime of strong local interaction breaks down. In this regime, coarsening is observed, where smaller clusters grow through coagulation events. We provide numerical experiments with a positivity-preserving numerical scheme for an SPDE of Dean-Kawasaki type, consisting of the McKean-Vlasov equation and conservative noise.

Joint works with Rishabh Gvalani (ETH Zürich), Greg Pavliotis (Imperial London) and Anna Shalova (TU Eindhoven).

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## Finite speed of propagation for a class of stochastic thin-film equations

Günther Grün

We present an energy method to prove a qualitative result on finite speed of propagation for energy-dissipating solutions to stochastic thin-film equations with conservative nonlinear multiplicative noise under periodic boundary conditions. Physically, the equations under consideration correspond to weak slippage, i.e. the case of mobility exponents  $n \in (2, 3)$ . Analytically, our approach is based on two main ingredients. First, a new weighted decay estimate for appropriate powers of the solution which is set up in combination with Bernis inequalities and energy estimates. Secondly, tailored filtering techniques inspired by the methods which were presented for stochastic porous-medium equations with source-term noise in (J. Fischer and G. Grün, SIAM J. Math. Anal. 47:825-854, 2015).

This is joint work with L. Klein (Erlangen).

# **An additive-noise approximation to Keller–Segel–Dean–Kawasaki dynamics**

Adrian Martini

Technische Universität Berlin

In this talk, we consider the fluctuating hydrodynamics of a system in which particles are attracted to one another through a gravitational force (Keller–Segel dynamics). We propose an additive-noise approximation to the ill-posed Dean–Kawasaki equation and show that it retains the same law of large numbers and central limit theorem as (conjectured for) the particle system. We further deduce a large deviation principle and show that the approximation error lies in the skeleton equation that drives the rate function. Interspersed throughout the talk, we will consider open questions about both the Keller–Segel particle system and the associated fluctuating hydrodynamics. Based on joint work with Avi Mayorcas.

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## **Learning macroscopic evolution from particle dynamics**

Dirr, Nicolas

Cardiff University

If we are interested in the behaviour of a stochastic system on an intermediate or large scale, it is often not efficient to compute the evolution of the entire microscopic particle dynamics. Instead, one would like to have a PDE or stochastic PDE suitable for the chosen scale of interest. Using additional information from the thermodynamic structure of the system for learning the PDE turns out to have advantages over going for the operator directly. In addition to stability and a selection principle in case of non-uniqueness, we get an SPDE from fluctuating hydrodynamics and large deviations rate functionals. The key is observing fluctuations for determining the operator which defines the dissipative part of the evolution in the GENERIC framework.

Joint work with Celia Reina, Johannes Zimmer and students: Peter Embacher, Zequn He, Shenglin Huang, Xiaoguai Li.

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# **A quantitative central limit theorem for the simple symmetric exclusion process**

Vitalii Konarovskyi

Universität Hamburg

We will discuss a quantitative central limit theorem for the simple symmetric exclusion process (SSEP) on a multidimensional discrete torus. The argument is based on the comparison of the generators of the density fluctuation field of the SSEP and the generalized Ornstein-Uhlenbeck process, as well as on an infinite-dimensional Berry-Essen bound for the initial particle fluctuations. In turn, this relies on regularity estimates for the Ornstein-Uhlenbeck process, and higher order correlations of the SSEP. The obtained rate of convergence is optimal. It is a joint work with Benjamin Gess.

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## **ultiscale analysis for reversible diffusions**

Pavliotis, Greg

Imperial College

We consider the problem of periodic reiterated homogenization for a reversible diffusion in  $(N + 1)$ -scale locally periodic potential. We show that reversibility is preserved under homogenization. Furthermore, we obtain formulas for the effective mobility matrix and the coarse-grained free energy. We also explore links between this homogenization problem and the problem of identifying the optimal diffusion process (in the sense of maximizing the spectral gap, subject to appropriate constraints) among all reversible diffusions with respect to given Gibbs measure.

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# Well-posedness for Dean-Kawasaki models of Vlasov-Fokker-Planck type

Max von Renesse

Universität Leipzig

We consider systems of interacting particles which are described by a second order Langevin equation, i.e., particles experiencing inertia. We introduce an associated equation of fluctuating hydrodynamics, which can be interpreted as stochastic version of a Vlasov-Fokker-Planck equation. We show a dichotomy previously known for purely diffusive systems carries over to the hypoelliptic setting considered here: Solutions exist only for suitable atomic initial data, but not for smooth initial data. The class of systems covered includes several models of active matter.

Joint work with Fenna Müller and Johannes Zimmer.

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## A CLT and LDP for the generalised Dean–Kawasaki equation on a bounded domain

Shyam Popat

Oxford University

We begin with a short motivation on how the study of fluctuations of SPDEs are relevant when interested in fluctuations of particle systems. We then present some recent ongoing results on the central limit theorem and large deviations principle for the generalised Dean–Kawasaki SPDE. Our main contribution is the ability to consider the equation on a general  $C^2$ -regular, bounded domain with Dirichlet boundary conditions. On the particle level, the boundary condition corresponds to absorption and injection of particles at the boundary.

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# Higher order fluctuation expansions for nonlinear stochastic heat equations in singular limits

Zhengyan  
Wu

Higher order fluctuation expansions for stochastic heat equations (SHE) with nonlinear, non-conservative and conservative noise are obtained. These Edgeworth-type expansions describe the asymptotic behavior of solutions in suitable joint scaling regimes of small noise intensity and diverging singularity. The results include both the case of the SHE with regular and irregular diffusion coefficients. In particular, this includes the correlated Dawson-Watanabe and Dean-Kawasaki SPDEs, as well as SPDEs corresponding to the Fleming-Viot and symmetric simple exclusion processes. This is joint work with Benjamin Gess and Rangrang Zhang.

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## Deriving a thermodynamic system from a Hamiltonian one

Johannes Zimmer  
Technische Universität München

We reconsider the problem of coarse-graining a “microscopic” Hamiltonian dynamics which describes the motion of molecules to obtain a macroscopic system which includes dissipative mechanisms. In particular, we study the thermodynamical implications concerning Hamiltonians, energy, and entropy and the induced geometric structures. Randomness enters through the initial data of a heat bath component of the original Hamiltonian system.

The analysis uses some mathematical tools from the theory of dilations, which will be described at the hand of an example.

The main interest is, however, to analyse the resulting system in the formulation of GENERIC (General Equations for Non-Equilibrium Reversible Irreversible Coupling). We will show that the coarse-grained version has this structure, with conserved energy, nondecreasing entropy and an Onsager operator describing the dissipation. A particular interest will be to understand how the building blocks of GENERIC arise from the microscopic Hamiltonian system. In terms of fluctuating hydrodynamics, this includes an analysis how noise in the initial data of the underlying Hamiltonian system results in “stochastic GENERIC”.

This is joint work with Alex Mielke and Mark Peletier.

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