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Chair of Optimal Control

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Optimal Control:
Identify control variables in a physical system, such that a desired outcome is reached, or the outcome is close to measured data.

Infinite dimensional optimal control problem:

$$\min_{q \in Q, u \in U} J(q,u) \text{ s.t. } F(q,u) = 0$$

- $F(q,u) = 0$ encodes a PDE (partial differential equation) with solution u and control q
- $u = u(x)$, $q = q(x)$ are functions, depending on $x \in \Omega$, Q, U are subsets of function spaces
- $J(q,u)$ measures how close q and u are to the desired outcome

Modelling/Analytical Aspects:

Find $J(q,u)$ and $F(q,u)$, which mathematically describe the problem, and show **existence** of a solution to the PDE and the optimal control problem and additional analytical properties.

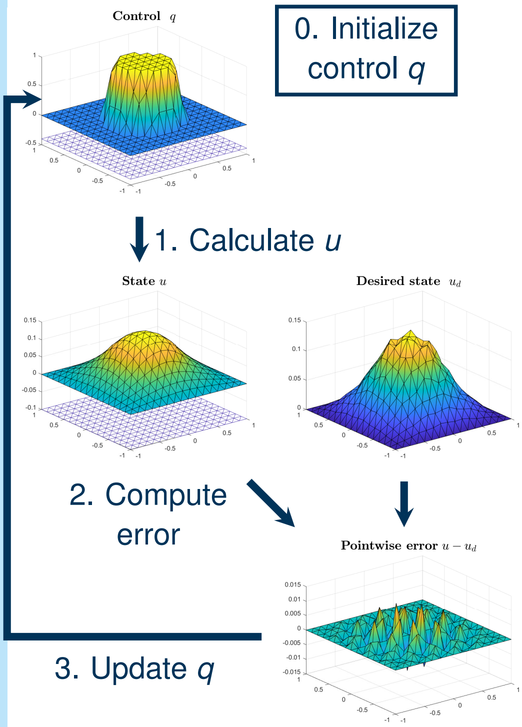
Typical Questions:

- Does a unique solution of the PDE exist?
- Does there exist an optimal pair $(\bar{q}, \bar{u})$, i.e. $J(\bar{q}, \bar{u}) \leq J(q,u)$ for all $(q,u) \in Q \times U$?
- Can we find conditions that tell us, if a pair (q,u) is the optimum?
- Can we devise algorithms, that find pairs (q,u) which fulfill these conditions?

$\Rightarrow$ Optimization methods in function space

Example:

$$\min_{q,u} \frac{1}{2}\|u - u_d\|^2_{L^2(\Omega)} + \frac{\alpha}{2}\|q\|^2_{L^2(\Omega)}$$
$$\text{s.t. } -\Delta u + u - q = 0 \text{ in } \Omega$$
$$u = 0 \text{ on } \partial\Omega$$



Numerical Aspects:

Computers cannot deal with an infinite dimensional problem, so we need to approximate it by a finite dimensional one. We only calculate approximations of the PDE solution at discrete points.

Typical Questions:

- How do we create our mesh?
- Which ansatz functions do we use to approximate q and u? (e.g. finite elements, neuronal networks, finite volume, ...)
- How fast does the error decrease, depending on the mesh size?
- How can we make sure, the optimization algorithm takes the same number of iterations, independently of how fine we discretize the PDE? (= Mesh independence)

Possible Applications:

- Which shape (q) does the wing of an airplane need to have, such that the air flowing over it (u) creates the largest possible lift force (J)?
- Where is the gas leak (q) that creates the pollution (u) that fits a measured one best (J)?
- Which blood pressure (q) creates a bloodflow (u) that is as close as possible (J) to a measured flow?