Technical University of Munich

Summer Workshop in the Calculus of Variations

Organized by
Luca Briani, Munich
Marco Cicalese, Munich
Leonard Kreutz, Munich

July 20th – July 23rd, 2026

Table of Contents

Roberto Alicandro
TBD 3

Paolo Bonicatto
On the Geometric Transport Equation for currents 3

Andrea Braides
Homogenization of Hamilton-Jacobi equations with defects 3

Camilla Brizzi
On the q-integrability of the p-Wasserstein barycenters 4

Giacomo Canevari
A "not so strange" term in the homogenisation of a problem with Robin boundary conditions 4

Davide Carazzato
A strong quantitative isoperimetric inequality for a capillarity problem . 4

Vito Crismale
Quasistatic cohesive-type crack growth 5

Lucia De Luca
Asymptotic expansion of the relative s-fractional perimeter as s tends to 0 5

Matteo Focardi
Free Discontinuity problems: from Griffith's energy to cohesive models . 5

Tobias König
Stability of Sobolev-type inequalities: best constants and optimizers 6

Carolin Kreisbeck
Functions with zero nonlocal gradient and their role in nonlocal Neumann-type problems 6

Andrea Kubin
Consistency of Some Variational Schemes for Modeling Geometric Flows 6

Thomas Leblé
Local optimality of the hexagonal lattice 7

Andrea Marchese
Lipschitz solutions to prescribed gradient, divergence and Jacobian for singular measures 7

Luc Nguyen
Mountain pass for the Ginzburg-Landau energy in a strip: solitons and solitonic vortices 7

Anastasija Pešić
TBD 7

Remy Rodiac
Convergence of critical points of the Ambrosio-Tortorelli functional with convergence of the phase-field energy 8

Pascal Steinke
Second-Order Gamma-limit for the Cahn-Hilliard Functional 8

Antonio Tribuzio
Qualitative and quantitative analysis of a T3 structure in linearized elasticity 8

Konstantinos Zemas
From 0.01% *to* 70.72% *validity of Ward's conjecture on Hopfions* 9

Abstracts

TBD

Roberto Alicandro

On the Geometric Transport Equation for currents

Paolo Bonicatto

I will report on recent results concerning the Geometric Transport Equation for k-dimensional currents in $\mathbb{R}^n$. This equation generalises the classical continuity and transport equations to model the motion of geometric objects such as lines and surfaces. I will discuss well-posedness results for Lipschitz vector fields, highlighting a deep connection with the decomposability bundle of a measure introduced by Alberti and Marchese. This theory further extends to the time-dependent setting with minimal regularity in time of the vector field, thus offering a unified framework for the evolution of geometric data under non-smooth flows. If time allows, I will also outline a recent approach to the classical Frobenius' theorem via the transport of currents. The vanishing bracket condition is recast into transport identities that remain meaningful even when one of the vector fields is a normal 1-current and this perspective sheds light on some Alfvén-type statements in magnetohydrodynamics.

Homogenization of Hamilton-Jacobi equations with defects

Andrea Braides

We consider a variational approach to the homogenization of periodic H-J equations, when perturbed by a local potential. By the use of the Lax-Hopf formula we show a stability result under very weak conditions for positive perturbations, and that stability never occurs if the perturbation is negative. For viscous H-J equations the Lax-Hopf formula must be suitably stated in a probability space, and a new type of random-to-deterministic homogenization must be used. This allows some stability results, but not as fine as in the non-viscous case. Work in collaboration with G.Dal Maso and C. Le Bris, after earlier results in terms of viscosity solutions by PL Lions, P. Souganidis, and others

On the q-integrability of the p-Wasserstein barycenters

Camilla Brizzi

In this talk we discuss the L^q-regularity of the density of barycenters of N probability measures with respect to the p-Wasserstein metric ($p > 1$). According to a previous result, if one marginal is absolutely continuous, so is the W_p-barycenter. The next natural question is whether the L^q-regularity of the density of the first marginal is also preserved for any $q > 1$, as it happens in the classical case ($p = 2$) of Agueh–Carlier, or for W_p-geodesics ($N = 2$). Here we show that this is not the case: as soon as $N > 2$ and p different from 2, it is possible to find examples of W_p-barycenters which are not q-integrable, even if one marginal is compactly supported and bounded. In particular, L^q-regularity is preserved only if the supports of all the marginals satisfy suitable geometric assumptions. However one can see that in dimension 1, if the densities of all the marginals (not only of one) are in L^q, the W_p inherits the same integrability, without requiring any additional geometrical assumptions. In case of higher dimension this question is still open. Finally, we provide some examples where the W_p-barycenter can be explicitly computed and it is regular.

A "not so strange" term in the homogenisation of a problem with Robin boundary conditions

Giacomo Canevari

We consider Laplace's equation in a periodically perforated domain, with Robin boundary conditions on the holes and a Robin coefficient inversely proportional to the total surface area of the holes. We show that, in a critical regime, the homogenised equation contains an additional zero-order term, which is defined in terms of a suitable eigenvalue problem and depends nonlinearly on the Robin coefficient. As the latter tends to infinity, the additional term converges to the capacitary "strange term" found by Cioranescu and Murat in the homogenisation of a problem with Dirichlet boundary conditions. This talk is based on joint work with K. Cherednichenko (University of Bath) and A. Zarnescu (BCAM, Bilbao, and "Simion Stoilow" Institute of the Romanian Academy).

A strong quantitative isoperimetric inequality for a capillarity problem

Davide Carazzato

During the seminar, we will introduce a stronger version of the quantitative isoperimetric inequality, originally developed by Fusco and Julin, where the distance from the optimizers is computed in terms of the oscillation of the normal. We obtain a quantitative inequality for the capillarity perimeter in the halfspace in terms of a similar asymmetry using the so-called selection principle, based on the regularity theory for the perimeter functional. We present the fundamental ideas, emphasizing the differences between Fusco and Julin's problem and ours. This result was obtained in collaboration with Giulio Pascale and Marco Pozzetta.

Quasistatic cohesive-type crack growth

Vito Crismale

With Manuel Friedrich (JKU Linz), we proved the existence of globally stable quasistatic evolutions for a cohesive-type fracture model in the SBV framework. The crack path is not prescribed a priori nor subject to any topological restriction, and the result holds in any dimension. This generalizes to a cohesive setting the existence result by Francfort and Larsen in 2003.

Asymptotic expansion of the relative s-fractional perimeter as s tends to 0

Lucia De Luca

We prove a first-order Gamma-convergence result (as s tends to 0) for the s-fractional perimeter relative to a given bounded set and with prescribed exterior datum. We investigate the robustness of our approach under perturbations of the exterior datum and in presence of volume constraints. We then use the Gamma-convergence expansion to study the asymptotic behavior of s-minimal sets as s tends to 0. Joint work with G. Palatucci, M. Piccinini, and M. Ponsiglione.

Free Discontinuity problems: from Griffith's energy to cohesive models

Matteo Focardi

Free-discontinuity energies whose bulk and surface densities exhibit superlinear growth, respectively for large gradients and small jump amplitudes, emerge in various fields, for instance in Imaging and in Fracture Mechanics.

A distinctive feature of this kind of models is that the functionals are defined on $(G)SBV$ functions whose jump sets may have infinite measure. Establishing general lower semicontinuity and relaxation results in this setting requires new analytical techniques. In addition, variational approximation of superlinear energies via phase field models is key to the application of numerical methods.

The talk is based on joint work with Sergio Conti (U. Bonn) and Flaviana Iurlano (U. Genova).

Stability of Sobolev-type inequalities: best constants and optimizers

Tobias König

The well-known stability inequality due to Bianchi and Egnell (1991) bounds the deficit of the Sobolev inequality from below by a non-explicit positive constant c_{BE} times the square of the $\dot{H}^1(\mathbb{R}^d)$-distance to the set of Aubin-Talenti bubbles to the power two. While the power two is optimal, it is a major open question to determine the sharp value of c_{BE} and the corresponding optimizer.

In this talk I will present an overview of recent partial results in that direction, in particular concerning the attainability of the best constant c_{BE}. The findings display a rich behavior which depends in a remarkable way on the space dimension $d \geq 1$ and the fractional order $s > 0$ of the Sobolev inequality. Surprisingly, in the setting of the reverse Sobolev inequality corresponding to $s > d/2$, the value of the constant c_{BE} can be explicitly determined for certain values of s and d.

Functions with zero nonlocal gradient and their role in nonlocal Neumann-type problems

Carolin Kreisbeck

In this talk, we explore functions whose nonlocal gradient, more precisely, finite-horizon fractional gradient, vanishes. Surprisingly, unlike in the classical local theory, this class forms an infinite-dimensional vector space. Our main result characterizes functions with zero nonlocal gradient in terms of two simple features: their values in a boundary layer and their mean value. The proof relies on recent advances in the solution theory for pseudo-differential boundary-value problems. We also discuss regularity properties and present illustrative examples. As an application, we provide new tools for nonlocal Sobolev spaces without complementary-value conditions, including Poincaré inequalities and compactness results obtained by factoring out functions with vanishing nonlocal gradient. These results can be used to study nonlocal partial differential equations with natural boundary conditions, in particular nonlocal Neumann-type problems. In this setting, we establish well-posedness and prove convergence to the corresponding local problems via Γ-convergence as the fractional parameter tends to 1. This is joint work with Hidde Schönberger (UC Louvain).

Consistency of Some Variational Schemes for Modeling Geometric Flows

Andrea Kubin

In this talk, I will discuss variational schemes for modeling surface diffusion in three dimensions, anisotropic surface diffusion with elasticity in two and three dimensions, and volume-preserving anisotropic mean curvature flow in two dimensions. I will focus in particular on the consistency of these schemes. The results

presented are drawn from several joint works with M. Cicalese, N. Fusco, V. Julin, A. Kubin, D. A. La Manna, and E. Pasqualetto.

Local optimality of the hexagonal lattice

Thomas Leblé

If one tries to arrange 1 euro coins on a table in the most compact way, one should place the centers of the coins on the sites of an hexagonal lattice. It is believed that the hexagonal lattice also gives the optimal configuration of points for a whole family of energy minimization problems, this is the Cohn-Kumar "universal optimality" conjecture. I will present what is known about this question, including a recent **local** optimality result.

Lipschitz solutions to prescribed gradient, divergence and Jacobian for singular measures

Andrea Marchese

In this talk I will explain how to find Lipschitz solutions in a Lusin-type sense with respect to singular measures to prescribed gradient, divergence or Jacobian determinant, with merely bounded data, seemingly in contrast with the philosophy behind Ornstein's non-inequalities. I will also discuss connections with the structure of metric currents.

Mountain pass for the Ginzburg-Landau energy in a strip: solitons and solitonic vortices

Luc Nguyen

We study critical points of the Ginzburg-Landau energy in an infinite strip where phase imprinting is applied to half of the domain. We prove that there is a critical width of the cross section below which the soliton solution is a mountain pass solution and the minimizer within the subspace of odd functions. Above the critical width, we find that the mountain pass solution is a vortex with a solitonic behaviour in the infinite direction, called a solitonic vortex. Moreover, depending on the width, we prove that the minimizer in a space with some symmetries can display one or several solitonic vortices. Time permitting, we will discuss generalizations and open questions for the case of an infinite cylinder in 3D. Joint work with Amandine Aftalion.

TBD

Anastasija Pešić

Convergence of critical points of the Ambrosio-Tortorelli functional with convergence of the phase-field energy

Remy Rodiac

In order to describe the behaviour of an elastic material undergoing fracture we can use a variational model and the so-called Mumford-Shah energy defined on a subspace of SBV functions. One difficulty is that critical points of this energy are difficult to approximate by numerical methods. One can then think of approximating the Mumford-Shah energy by another energy defined on a space of more regular functions (H^1-functions): the Ambrosio-Tortorelli energy. It is known since the pioneer work of Ambrosio-Tortorelli that (global) minimizers of this energy converge towards minimisers of the Mumford-Shah energy. In this talk we will show that under an assumption of convergence of the energies, critical points of the Ambrosio-Tortorelli energy also converge to critical points of the Mumford-Shah energy. In 2d, we can relax the convergence of the energies and assume only the convergence of the phase-field energies. This is based on joint works with Jean-François Babadjian and Vincent Millot, and with Jean-François Babadjian and Martin Rakovsky.

Second-Order Gamma-limit for the Cahn-Hilliard Functional

Pascal Steinke

We discuss a higher-order Gamma-convergence result for the Cahn-Hilliard functional which describes the separation of fluids. After a short introduction to the general framework of higher-order Gamma-convergence, we are going to discuss our main result about the second-order Gamma-limit in the case of Dirichlet boundary data with subquadratic growth at the wells. This is a joint work with F. Colasanto

Qualitative and quantitative analysis of a T3 structure in linearized elasticity

Antonio Tribuzio

In this talk, we study the rigidity properties of a differential inclusion in linearized elasticity. We will introduce and discuss a set K of three diagonal strains that are pairwise incompatible but with non-trivial (symmetrized) rank-1-convex hull. This is called a T3 structure. We first prove that K is rigid at the level of exact solutions, namely that Lipschitz maps whose gradient is locally in K must be affine. After this, we study the scaling behaviour of the corresponding singularly-perturbed elastic energy, giving quantitative information on the flexibility of K in the sense of approximate solutions.

From 0.01% to 70.72% validity of Ward's conjecture on Hopfions

Konstantinos Zemas

The Faddeev-Skyrme model is a by now classical nonlinear $O(3)$-sigma model which has proved very successful in quantum field theory, in particular in detecting topological solitons. A prototype mathematical model for fields $u\colon \mathbb{S}^3 \to \mathbb{S}^2$ consists of the harmonic map energy together with its symplectic variant acting as a singular perturbation. We prove that, modulo rigid motions, the Hopf map is the unique minimizer of the Faddeev–Skyrme energy in its homotopy class, for an explicit sufficiently large range of the coupling constant.

This is joint work with A. Guerra (University of Cambridge) and X. Lamy (University of Toulouse).