Technical University of Munich

Frontiers in Calculus of Variations and Applied Analysis

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Abstracts

Frobenius theorem, contact sets, and the locality of curl

Giovanni Alberti
(joint work with P. Bonicatto, G. Del Nin, A. Massaccesi and A. Merlo)

Frobenius theorem states that there exist no k-dimensional surface S which is tangent to a non-involutive distribution of k-planes V. One may wonder if this statement holds in weaker contexts; in particular, following Z. Balogh and S. Delladio, we ask if every contact set E has null k-dimensional volume (a contact set of the surface S and the distribution V is a subset E of S such that S is tangent to V at every point of E). The complete answer depends on a combination of the regularity of S and of the boundary of E: at one end of the spectrum, if S is of class $C^{1,1}$ then the answer is positive regardless of the regularity of the boundary of E; at the other end, if S is of class C^1 then the answer is positive only for sets E in a certain fractional Sobolev class. This question can be rephrased in terms of locality of the curl operator in dimension two: given a vector field u in L^1 whose distributional curl $curl(u)$ belongs to L^1 and a set E set such that $u = 0$ a.e. on E, is it true that $curl(u) = 0$ a.e. on E? Again, the answer depends on a combination of the regularity of u and of the boundary of E (note by comparison that if we replace the curl by the gradient, the answer is positive with no further assumption on u and E). These results are part of a research project involving Paolo Bonicatto (University of Trento), Giacomo Del Nin (MPI-MIS, Leipzig), Annalisa Massaccesi (University of Padova) and Andrea Merlo (University of the Basque Countries).

Boundary defects in Liquid Crystal

Lia Bronsard
(joint work with S. Alama, A. Colinet, S. Locke, D. Louizos, D. Stantejsky and L. van Brussel)

We study the effect of “weak” and “strong” boundary conditions on the location and type of defects observed in a Landau de Gennes thin-film model for liquid crystals. We study the minimizers of the associated Ginzburg-Landau energy as well as its Gamma limit when the correlation length tends to zero. A-priori estimates in the case that splay and bend moduli are included in the energy will also be presented as well as results in the case of the 3D Landau-de Gennes model with a magnetic field. These represent joint works with S. Alama, A. Colinet, S. Locke, D. Louizos, D. Stantejsky and L. van Brussel.

Optimal domains for the Cheeger inequality

Giuseppe Buttazzo

We study a generalized form of the Cheeger inequality by considering the shape functional $F_{p,q}(\Omega) = \lambda_p^{1/p}(\Omega)/\lambda_q^{1/q}(\Omega)$, where the original Cheeger case corresponds to $p = 2$ and $q = 1$. Here $\lambda_p(\Omega)$ denotes the principal eigenvalue of the Dirichlet p-Laplacian. The infimum and the supremum of $F_{p,q}$ are discussed, together with the existence of optimal domains. Some open problems will be illustrated as well.

A simpler characterization of GBD

Antonin Chambolle

(joint work with V. Crismale)

The space GBD, introduced by G. Dal Maso to address variational problems in fracture mechanics, is characterized by slicing properties of the displacements, and a control of the $1D$ slices in all directions, which is not natural in this context. We give a simple proof that controlling $d(d + 1)/2$ directions in dimension d is sufficient to characterize properly GBD displacements. This is joint with Vito Crismale (Roma La Sapienza).

Long-time behaviour of damped adhesive strings

Giuseppe Maria Coclite

(joint work with N. De Nitti, F. Maddalena, G. Orlando and E. Zuazua)

The talk will focus on an evolutionary PDE modelling a one-dimensional linearly elastic body interacting with a rigid substrate through an adhesive material. More precisely, the PDE is a damped semilinear wave equation with Neumann boundary conditions. The main feature of the equation is the nonlinear force acting on the body: it is proportional to the displacement for small displacements and it is zero when the displacement is large. The talk will be divided in three parts. In the first part, we will recall some key aspects of the classica theory of maximal monotone operators to deduce the global well-posedness for the model. In the second part, we will focus on the qualitative asymptotic behaviour of solutions for large time, showing that solutions converge to solutions of the steady problem. Finally, in the third part, we will show how to deduce a quantitative estimate on the rate of convergence to the asymptotic limit. The content of the talk is based on a paper in collaboration with N. De Nitti, F. Maddalena, G. Orlando, E. Zuazua

Non-unique vanishing viscosity solutions to the forced 2D Euler equations

Maria Colombo

The talk will provide an overview of the developments on non-uniqueness of solutions of the Euler and Navier-Stokes equations, focussing on results about the sharpness of Yudovich well-posedness theorem for the 2 dimensional Euler equations. It will then highlight new results showing that solutions obtained in the vanishing viscosity limit from the (well-posed) Navier-Stokes equations can be nonunique.

Variational phase-field models of cohesive fracture

Sergio Conti
(joint work with M. Focardi and F. Iurlano)

Cohesive-zone models are free-discontinuity problems with a surface energy that converges to zero for small values of the jump, and to a positive, finite value for large values of the jump. Their approximation with a phase-field model requires an interaction between the damage variable and the strain that depends on the regularization parameter. I will discuss recent progress on the development of these approximations and on the study of the limiting model. An important technical step is a finite-element approximation of SBV functions with jump set of infinite length. The talk is based on joint work with Flaviana Iurlano and Matteo Focardi.

Continuous symmetry breaking in an isoperimetric problem with competing long-range interactions

Sara Daneri

We introduce a rigorous approach to the study of the symmetry breaking and pattern formation phenomenon for isotropic functionals with local/nonlocal interactions in competition. We consider a general class of nonlocal variational problems in arbitrary dimension in which an isotropic surface term favouring pure phases competes with an isotropic nonlocal term with power law kernel favouring alternation between different phases. Close to the critical regime in which the two terms are of the same order, we give a rigorous proof of the conjectured structure of global minimizers, in the shape of periodic domains with flat boundary (e.g. stripes or lamellae).

The liquid drop model

Rupert Frank

The liquid drop model was originally introduced in the nuclear physics literature in 1930 and has recently been studied extensively using techniques from the calculus of variations, geometric analysis, PDE and mathematical physics. The talk will discuss some new results and open problems concerning a certain isoperimetric-type question in this model. In addition, we present a first rigorous analysis of a nuclear pasta phase encountered in astrophysics.

Linearization of variational models in brittle fracture

Manuel Friedrich
(joint work with P. Steinke and K. Stinson)

In this talk, I present a linearization result for Griffith functionals of brittle fracture. By means of Gammma-convergence we derive effective linearized functionals in the limit of vanishing strains, where the resulting free discontinuity problem in linear elasticity is formulated in the space of generalized special functions of bounded deformation. Based on this, we also discuss a result in the evolutionary framework, namely the convergence of energetic solutions describing quasistatic crack growth in the nonlinear and linearized setting. Joint work with P. Steinke and K. Stinson.

Mass splitting in the generalized Euler equations: a new explanation via discretization

Gero Friesecke

Arnold made the celebrated observation that solutions to the incompressible Euler equations of fluid dynamics correspond to geodesics in the group of volume-preserving diffeomorphisms. A nontrivial fact is that minimizers of the corresponding variational principle may not exist. Brenier introduced a relaxed variational formulation which he showed to be well-posed. Physically this formulation allows mass splitting, i.e., a fluid particle can move from A to B via an ensemble of trajectories. Mathematically this formulation is an instance of multi-marginal optimal transport; for a simple introduction to this formulation see my recent textbook on optimal transport [1]. After revieving the different formulations of the Euler equations, we - show that mass splitting still occurs after discretizing Brenier's relaxation in space and time - provide a short, transparent argument for the mass splitting which is much simpler than previous analyses of the continuous case. We close with a brief discussion from a modeling point of view: what is physically more correct, Euler (no mass splitting) or Brenier (mass splitting)?

[1] G. Friesecke, Optimal Transport: a Comprehensive Introduction to Modeling, Analysis, Simulation, Applications, SIAM, 2025, Link

Consistency for the surface diffusion flow in three dimensions

Nicola Fusco

We will discuss the flat flow solution for the surface diffusion equation via a discrete minimizing movements scheme proposed in 1994 in a celebrated paper by J.W. Cahn and J.E. Taylor. We will show that in dimension three the scheme converges to the unique smooth solution of the equation, provided the initial set is sufficiently regular.

Variational models for partial defects

Adriana Garroni

I will treat a class of sharp interface models for partial defects, in which partial defects are codimension 2 objects connected by codimension 1 objects. I will present two derivations of such energies. In dimension 2 a discrete model for crystal defects based on nearest neighbours and next to nearest neighbours interaction, via period potentials. In the asymptotic limit as the lattice spacing tends to zero, in terms of Gamma convergence, the model accounts for the formation and interaction of partial dislocations, point defects, and stacking faults, line defects connecting partials. The model falls into a class of discrete models with topological (fractional) singularities. One of the key ingredients is the characterisation of the minimisers for the core energy of the singularities, the so called one-vortex solutions. A second approach, strictly related, consists in a diffuse interface energy, which can be interprete as a semi-discrete model, à la Nabarro Peierls, which has the structure of a phase transition model with a multiple well potential and a non locale singular perturbation. This latter gives rise to the same asymptotics allowing also to treat the three dimensional case, under the assumption that the partial line dislocations lie on a single slip plane.

Curvature penalization of strongly anisotropic interface models and their phase-field approximation

Michael Goldman
(joint work with J.F. Babadjian and B. Buet)

In this talk I will present recent results obtained with J.F. Babadjian and B. Buet about the regularizing effects of curvature terms for interface models with strong anisotropy. We will consider two main (related) questions for two types of problems. The questions are lower semi-continuity of the energies and phase-field approximations. The models are isoperimetric problems on the one hand and free discontinuity problems on the other hand. Both are motivated by applications in material sciences. One of the original aspects of our work in the setting of free discontinuity problems, is the treatment of point energies which relies on a Gauss-Bonnet type result for varifolds.

Eventual regularity of the volume-preserving mean curvature flow in 3D

Vesa Julin
(joint work with V. Arya and S. Jeon)

I will consider flat flow solutions for the volume-preserving mean curvature flow constructed via the minimizing movements scheme. It is known that in 3D, such flows converge (up to translation of the components) to a union of spheres, and when the flow converges to a single sphere, this convergence is exponential. I will present a regularity result showing that, in the latter case, the flow eventually becomes smooth and converges exponentially fast in C^k-norm, for all k, to the sphere. A key technical novelty of this work is that we prove regularity without relying on monotonicity formulas. This is a joint work with Vedansh Arya (Jyvaskyla) and Seongmin Jeon (Seoul).

Higher Order Gamma Convergence

Giovanni Leoni
(joint work with I. Fonseca and L. Kreutz)

In this talk, we address the asymptotic development of order 2 by Gamma convergence of the Cahn-Hilliard functional with Dirichlet boundary conditions. This is joint work with I. Fonseca and L. Kreutz.

Peridynamics between Mechanics and Mathematics

Francesco Maddalena

Peridynamics is a nonlocal formulation in continuum mechanics, introduced by S. A. Silling (2000), which has attracted considerable interest in both the applied and theoretical fields. In this talk I will try to critically analyze the mechanical premises and the mathematical consequences of this model, in relation to the classical approach. In particular, I will try to investigate phenomena predicted and described by the nonlocal model, which escape the local model.

Elliptic regularisation of anisotropic flows

Massimiliano Morini
(joint work with C.A. Antonini, A. De Rosa and S. Stuvard)

After recalling a distributional formulation of anisotropic curvature flows, introduced some years ago to address the well-posedness of crystalline motions, we present an anisotropic variant of Ilmanen's elliptic regularization scheme. Under the non-fattening assumption, and by exploiting the aforementioned distributional formulation, we show the convergence of the scheme to the unique level set solution for a class of anisotropic motions. This is joint work with C.A. Antonini, A. De Rosa, and S. Stuvard.

Euler's elastica functional as a large mass limit of a two-dimensional non-local isoperimetric problem

Cyrill Muratov
(joint work with M. Novaga and T. Simon)

We consider the large mass limit of the non-local isoperimetric problem with a repulsive Yukawa potential in two space dimensions. In this limit, the non-local term concentrates on the boundary, resulting in the existence of a critical regime in which the perimeter and the non-local terms cancel each other out to leading order. We show that under appropriate scaling assumptions the next-order Γ-limit of the energy with respect to the L^1 convergence of the rescaled sets is given by a weighted sum of the perimeter and Euler's elastica functional, where the latter is understood via the lower-semicontinuous relaxation and is evaluated on the system of boundary curves. As a consequence, we prove that in the considered regime the energy minimizers always exist and converge to either disks or annuli, depending on the relative strength of the elastica term. This is joint work with M. Novaga and T. Simon.

Improving a Spectral Inequality by Payne

Carlo Nitsch

A celebrated inequality by Payne relates the first eigenvalue of the Dirichlet Laplacian to the first eigenvalue of the buckling problem. Motivated by the goal of establishing a quantitative version of this inequality, we show that Payne's original estimate–which is not sharp–can in fact be improved. Our result provides a refined spectral bound and opens the way to further investigations into quantitative enhancements of classical inequalities in spectral theory.

Lattice tilings with minimal perimeter and unequal volumes

Matteo Novaga

I will consider periodic tessellations of the Euclidean space with unequal cells, arising from the minimization of perimeter with volume constraint. I will present existence, regularity and qualitative properties of minimizers, discussing in particular the planar case, where explicit solutions can be found in some cases.

A singular perturbation problem for the Monge-Ampère equation

Heiner Olbermann
(joint work with P. Gladbach)

We consider the Monge-Ampère equation whose right hand side is a linear combination of delta distributions,

$$\mu_u = \sum_i c_i \delta_{a_i}$$

where μ_u denotes the Monge-Ampère measure of the unknown convex function u with zero boundary conditions, $c_i > 0$, and a_i are points in the domain for $i = 1, \ldots, m$. For suitable choices of c_i, a_i, the unique Alexandrov solution of this equation displays jumps of the gradient across (n-1)-dimensional simplices. We consider singular perturbations of this solution, via the the variational problem

$$\min_u \|\mu_u - \sum_i c_i \delta_{a_i}\|_{W^{-2,n}} + \varepsilon \|D^2 u\|_{L^2}$$

where $\varepsilon > 0$. The choice of this functional is motivated by a problem for 2-dimensional elastic thin sheets. We prove upper and lower bounds for this functional, that for the motivating example from nonlinear elasticity match up to logarithmic factors in ε, for small ε. In order to obtain the lower bound, we prove a generalization of the well known monotonicity of the subgradient that we consider of independent interest. Joint work with Peter Gladbach (formerly University of Bonn).

Monotonicity formulas for the inverse mean curvature flow and the *p*-Laplacian

Alessandra Pluda

In this talk we consider a family of monotonicity formulas which originate from seemingly different contexts in geometric analysis. On the one side, we have monotonicity formulas for the Laplace-Beltrami operator, tracing back to the work of Colding ("New Monotonicity Formulas for Ricci Curvature and Applications. I", 2012) and Colding-Minicozzi ("On Uniqueness of Tangent Cones for Einstein Manifolds", 2014). On the other side, we have the monotonicity of the Hawking mass along the inverse mean curvature flow, proved by Huisken and Ilmanen ("The Inverse Mean Curvature Flow and the Riemannian Penrose Inequality", 2001). The connection between these two formulas lies in monotonicity formulas of geometric quantities for the level sets of solutions to the p-Laplacian (formally the IMCF corresponds to the case $p = 1$). These general formulas require some regularity: we show that almost every level set is a curvature varifold for which a Gauss-Bonnet-type theorem is established. To prove the convergence to the inverse mean curvature flow we show that, as $p \to 1^+$, almost every level sets converge in the sense of curvature varifolds and gradients strongly converge in L^q for every finite q. Our monotonicity formulas imply several geometric inequalities, from the Willmore and the p-Minkowski inequalities to the Riemannian Penrose inequality.

The isoperimetric inequality for the capillary energy outside of convex sets.

Aldo Pratelli
(joint work with N. Fusco, V. Julin and M. Morini)

We consider the classical capillary isoperimetric problem outside of a convex set C; this means that we are given a parameter $-1 < \lambda < 1$ and the energy of any set E outside of C is given by $P(E) - (1+\lambda)\mathcal{H}^{N-1}(\partial E \cap \partial C)$. In words, the part of the boundary of E in C is "cheaper" than usual if $0 < \lambda < 1$, or it is "free" if $\lambda = 0$, or it has even a negative cost if $-1 < \lambda < 0$. If C is a half-space, then sets of minimal energy are simply spherical caps touching C with an angle $\theta = \arccos \lambda$. The case $\lambda = 0$ is the most studied, and basically everything is known, in particular the smallest possible minimal energy of sets of mass m is reached exactly in the case of the half-space. The main result that we will present is that the same is true also for any other value of λ. To obtain this result, we will use a special instance of the ABP-method; however, to make this method work, for the case $\lambda \neq 0$ we will need also a delicate argument of geometric type. Joint work with N. Fusco, V. Julin, M. Morini.

Quantitative estimates for the Dirichlet energy

Melanie Rupflin

In this talk we discuss the question of quantitative stability of minimisers for the classical Dirichlet energy of maps from R^2 into the unit sphere, i.e. whether, and with what rate, the distance of a map which almost minimise the energy (with given degree) to the nearest minimiser can be bounded in terms of the energy defect. We will see that there is a marked difference between maps of degree 1 and maps of higher degree and will discuss how a more flexible approach to quantitative stability and specially designed gradient flows can be used to establish sharp quantitative stability results for maps for which energy concentrates at multiple scales and/or near multiple points.

From discrete to continuum in the helical XY-model: emergence of chirality transitions

Francesco Solombrino
(joint work with M. Cicalese, D. Reggiani and M. Ruf)

We study the energy per particle of a ferromagnetic-anti-ferromagnetic frustrated spin chain with nearest and next-to-nearest interactions close to the Landau-Lifschitz point (where the helimagnetic-ferromagnetic transition occurs) , as the number of particles diverges. We rigorously prove the emergence of chiral ground states and we compute, by performing the Γ-limits of proper renormalizations and scalings, the energy for a chirality transition, if spins take value in the unit sphere S^1. Such a result does not hold if spins are S^2 valued, as in this case , as it is well established that in this case chirality transitions may emerge with vanishing energy. Inspired by recent work on the N-clock model, we consider a spin model where spins are constrained to a diverging number k_n of copies of S^1 covering S^2. We identify a critical energy-scaling regime and a threshold for the divergence rate of k_n, below which the -limit of the discrete energies capture chirality transitions while retaining an S^2-valued energy description in the continuum limit.

These are joint works with M. Cicalese, D. Reggiani and M. Ruf.

Energy of semi-coherent interfaces

Emanuele Spadaro

In this talk I will discuss models for analyzing inter-crystalline boundaries that arise from differences in atomic spacing. In the case of semi-coherent interfaces, where the misfit between crystal lattices is small, the interfaces can be resolved into sequences of edge dislocations, leading to an interfacial energy that exhibits ReadShockley-type superlinear scaling as a function of the misfit.

On the Rearrangement of functions and their applications

Cristina Trombetti

In this talk, we revisit classical results on the rearrangement of functions, highlighting their mathematical significance and structural properties. We further explore both traditional and recent applications, emphasizing the role of rearrangement techniques.