# Circuit cutting

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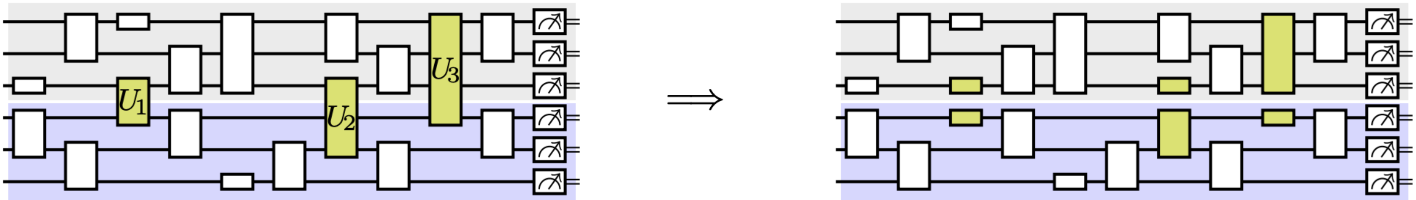



## Cost of gate cutting :

exponential in
number of cuts

## Results (informal)

Cutting $n$ - CNOT gates costs

$O(4^n)$ overhead

# Quasiprobability simulation

$$\underbrace{\mathcal{E}(\cdot)}_{\text{we want to perform}} = \sum_i \underbrace{a_i}_{\in \mathbb{R}} \underbrace{\mathcal{F}_i(\cdot)}_{\text{we can perform}}$$

QPD

# Expectation values :

$$\underbrace{\mathrm{tr}[M\,\mathcal{E}(\cdot)]}_{\text{want to compute}} = \sum_i a_i \overbrace{\mathrm{tr}[M\,\mathcal{F}_i(\cdot)]}^{\text{can compute}}$$

$$= \sum_i p_i \,\mathrm{tr}[M\,\mathcal{F}_i(\cdot)]\, \kappa \,\mathrm{sgn}(a_i)$$

$p_i = \frac{|a_i|}{\kappa}$ $\qquad$ $\kappa := \sum_i |a_i|$

- unbiased estimator
- larger variance $\rightarrow$ $\kappa^2$ - additional shots
- Total sampling overhead : $\prod_{i=1}^{n} \kappa_i^2 \rightarrow \exp(n)$
  (n-cuts)

Optimal decomposition

e.g. LO or LOCC

$$\gamma_S(\mathcal{E}) := \min_{a_i, \mathcal{F}_i} \left\{ \sum_i |a_i| : \mathcal{E}(\cdot) = \sum_i a_i \mathcal{F}_i(\cdot),\ a_i \in \mathbb{R}, \mathcal{F}_i \in S \right\}$$

# Applications of QPDs

| op. we want to run $\mathcal{E}(\cdot)$ | op. we can run $\mathcal{F}_i(\cdot)$ | known as |
|---|---|---|
| ideal (noise free) gate | noisy gates | PEC |
| arbitrary quantum gates | Clifford gates | classical sim algorithms |
| non-local gates (e.g. CNOT) | local gates [e.g. single-qubit gates] | circuit cutting |

I will focus on CNOT gate for rest of talk

Goal:

$$\gamma_{LO}(CNOT) = ?$$

$$\gamma_{LOCC}(CNOT) = ?$$

**Theorem:** $\gamma_{LO}(CNOT) = \gamma_{LOCC}(CNOT) = 3$

[Mitarai & Fujii, 2021]

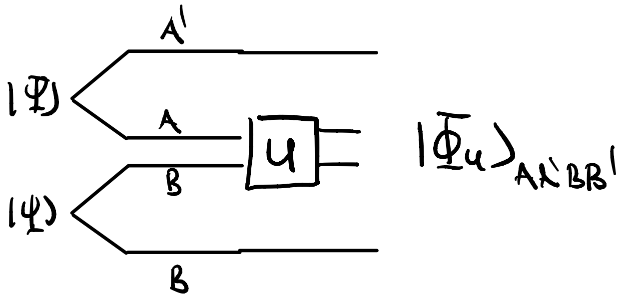
**Proof:**

① $\gamma_{LOCC}(CNOT) \leq \gamma_{LO}(CNOT) \leq 3$ : construct a basis that achieves $k=3$

② $\gamma_{LO}(CNOT) \geq \gamma_{LOCC}(CNOT) \geq 3$ :

Recall choi state of unitary $U_{AB}$

$$|\Phi_u\rangle_{AA'BB'} := (U_{AB} \otimes \mathbb{1}_{A'B'}) \left( |\Psi\rangle_{AA'} \otimes |\Psi\rangle_{BB'} \right)$$




$$\gamma_{LOCC}(U) \geq \underbrace{\gamma_{LOCC}(|\Phi_u\rangle\langle\Phi_u|)}_{\text{what is this?}} = 3$$

State preparation

$$S_{AB} = \sum_i a_i \, \sigma_A^{(i)} \otimes \omega_B^{(i)}$$

**Fact:**

$$\gamma_{LO}(S_{AB}) = \gamma_{LOCC}(S_{AB}) = 1 + 2E(S_{AB})$$

robustness of entanglement [Vidal & Tarrach 1999]

$E(|\psi\rangle) = \left(\sum_i \alpha_i\right)^2 - 1$

Schmidt coeffs

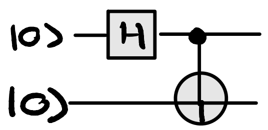
Is it cheaper to cut 2 CNOTs simultaneously than cutting twice a CNOT?

$$\gamma(\mathrm{CNOT}\otimes\mathrm{CNOT}) \leq \gamma(\mathrm{CNOT})\,\gamma(\mathrm{CNOT})$$

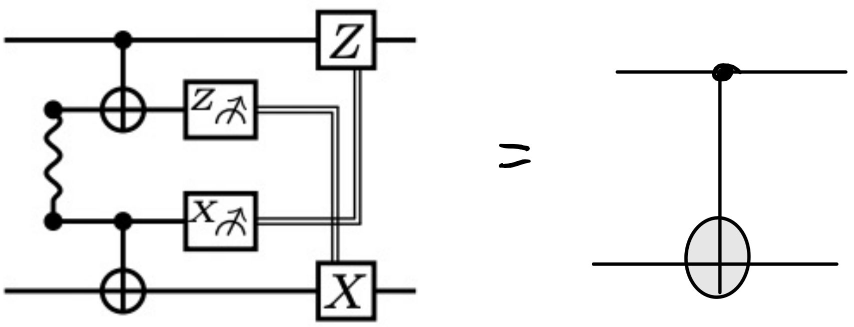
can it be strict ?

$\boxed{\text{LOCC}}$

Observation : $\boxed{1 \text{ ebit} \overset{\Delta}{\underset{\text{LOCC}}{=}} \mathrm{CNOT}}$

1) 

$|\Phi\rangle$ $\leq$

2) 

$\geq$

Bell states $|\Phi\rangle_{AB}$

$\gamma_{LOCC}(|\Phi\rangle\langle\Phi|_{AB}) = 3$ locality between $A\otimes B$

two-Bell states $A = A_1\otimes A_2$ , $B = B_1\otimes B_2$

$$\gamma_{LOCC}(|\Phi\rangle\langle\Phi|_{A_1B_1}\otimes|\Phi\rangle\langle\Phi|_{A_2B_2}) = 7 < 3^2 = 9$$

use entanglement between $A_1A_2$ and $B_1B_2$ !

$\longrightarrow$ preparing two Bell pairs simulteneously is cheaper than preparing a Bell state twice !

$$\left( \gamma_{LOCC} \left( |\Phi\rangle^{\otimes n} \right) \right)^{\frac{1}{n}} = \left( 2^{n+1} - 1 \right)^{\frac{1}{n}} \longrightarrow 2$$

## We understand :

1. Clifford gates
2. 2 qubit gates

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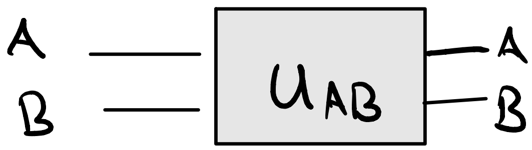
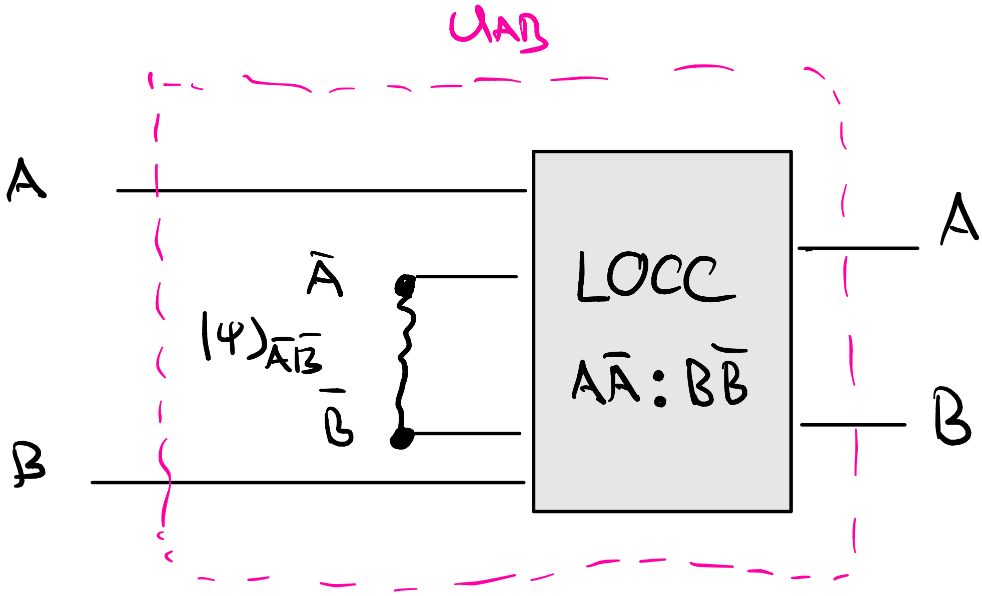
**Open question**

Gate teleportation for non-Clifford gates




What is <u>optimal</u> resource state $|\psi\rangle_{\bar{A}\bar{B}}$

- Clifford → Choi state
- non-Clifford → <u>not</u> Choi state