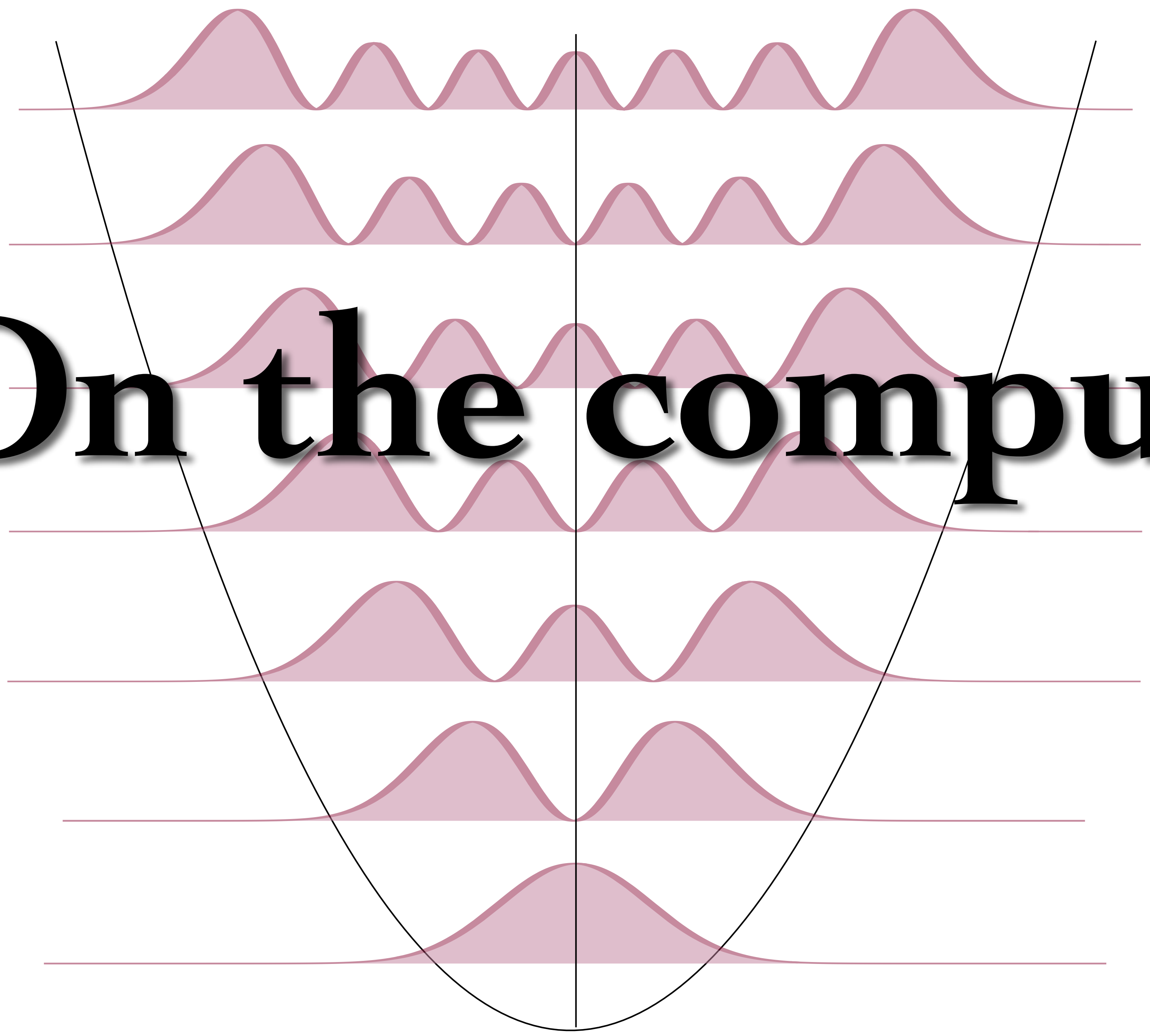


On the computational power of bosons

Ulysse Chabaud

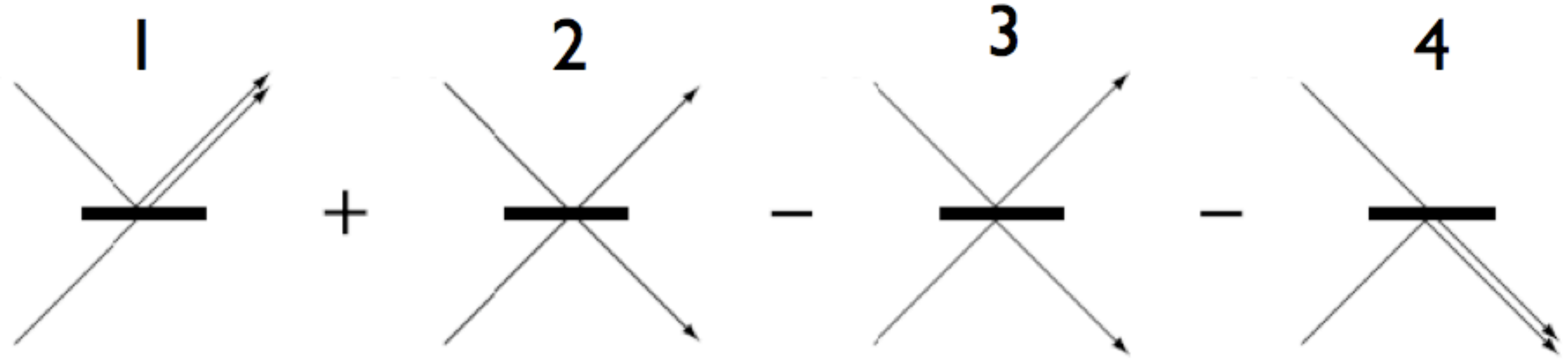
Context and motivation

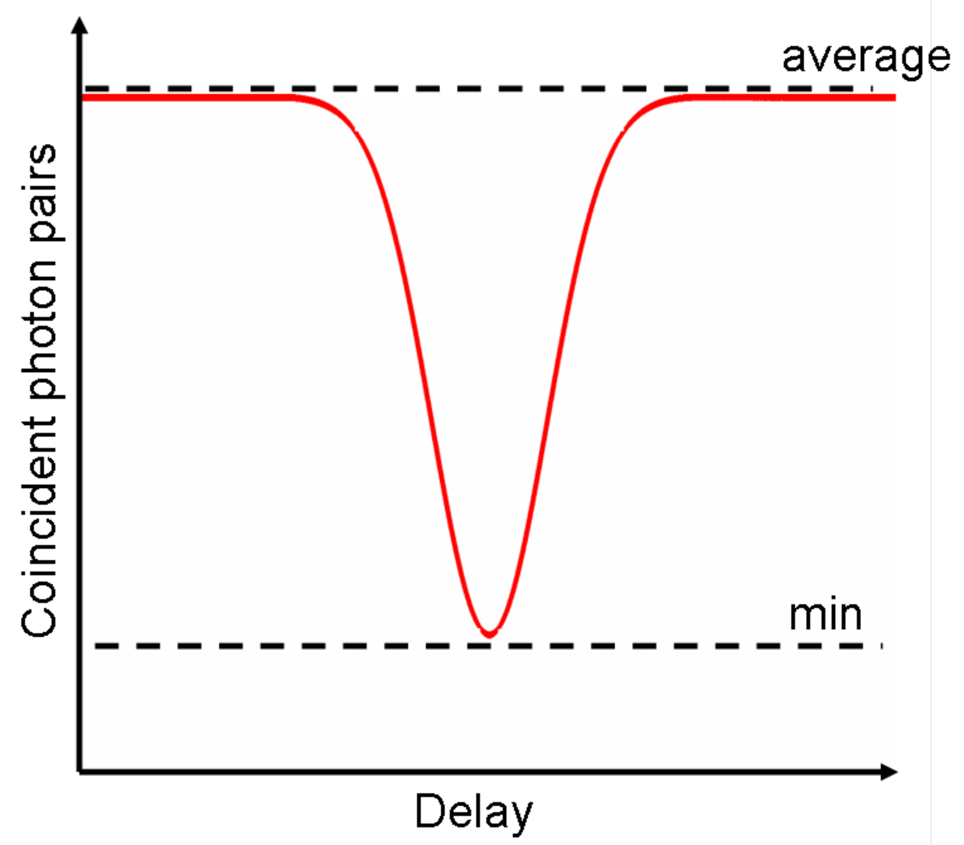
Bosonic systems

Context and motivation

Bosonic systems

- Bosonic statistics lead to remarkable physics



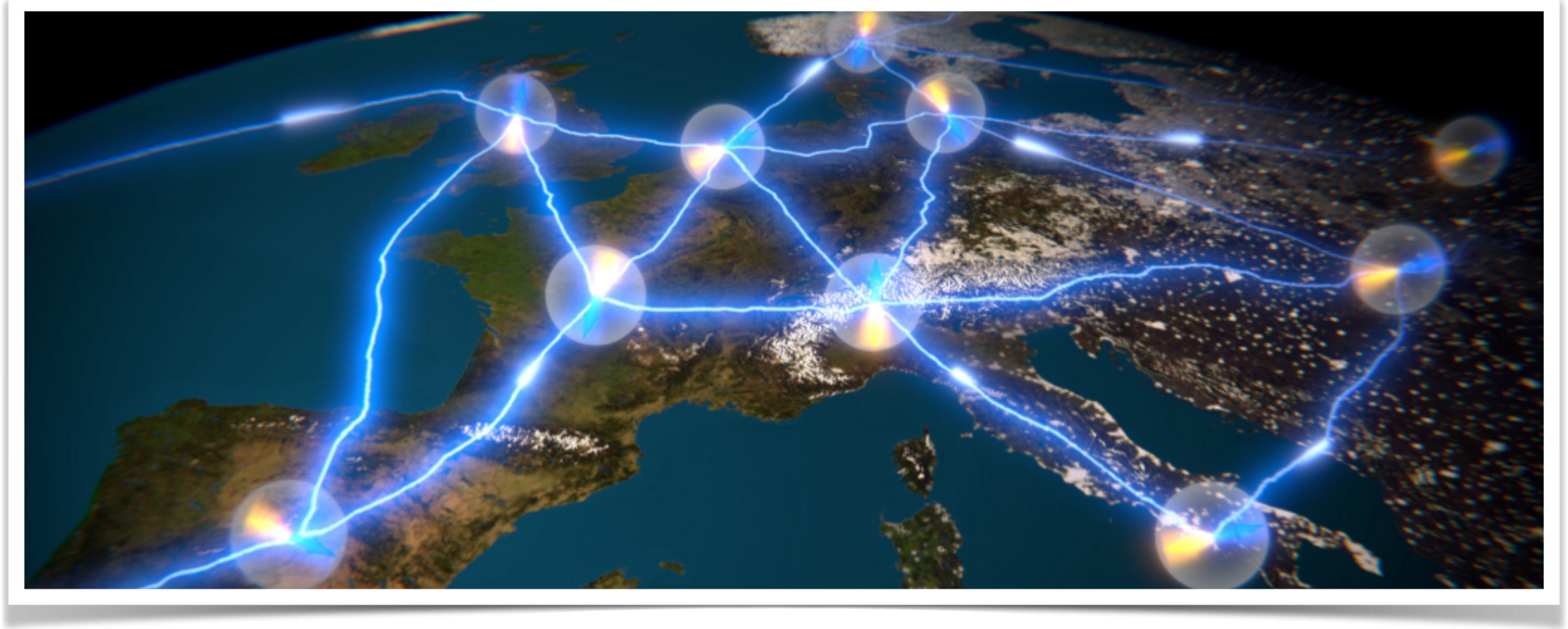


[Hong, Ou & Mandel, PRL 1987]

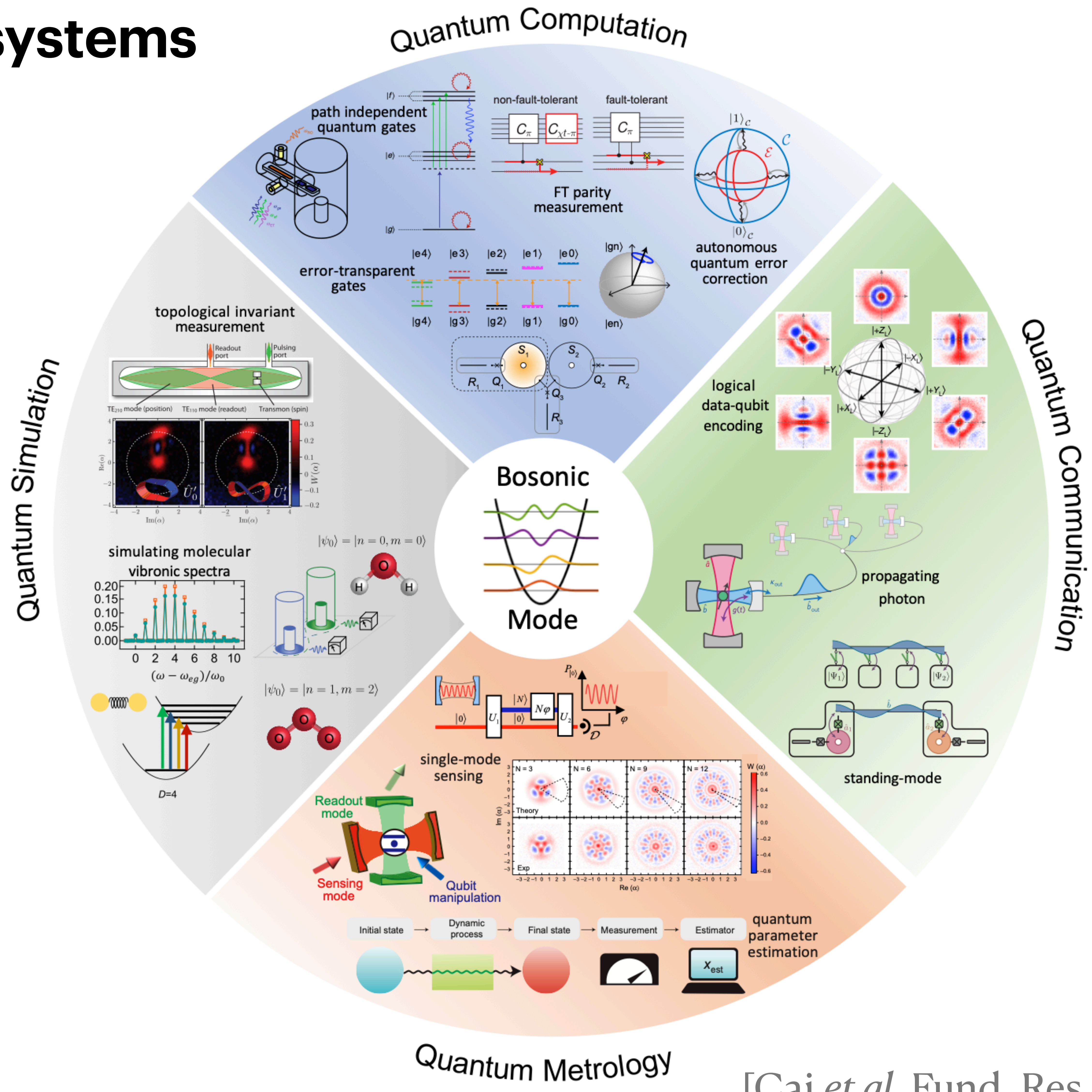
Context and motivation

Bosonic systems

- Bosonic statistics lead to remarkable physics
- Quantum light, superconducting bosonic modes…
- Crucial for quantum information processing



QUANTUM INTERNET ALLIANCE



[Cai *et al*, Fund. Res. 2021]

Context and motivation

Bosonic systems

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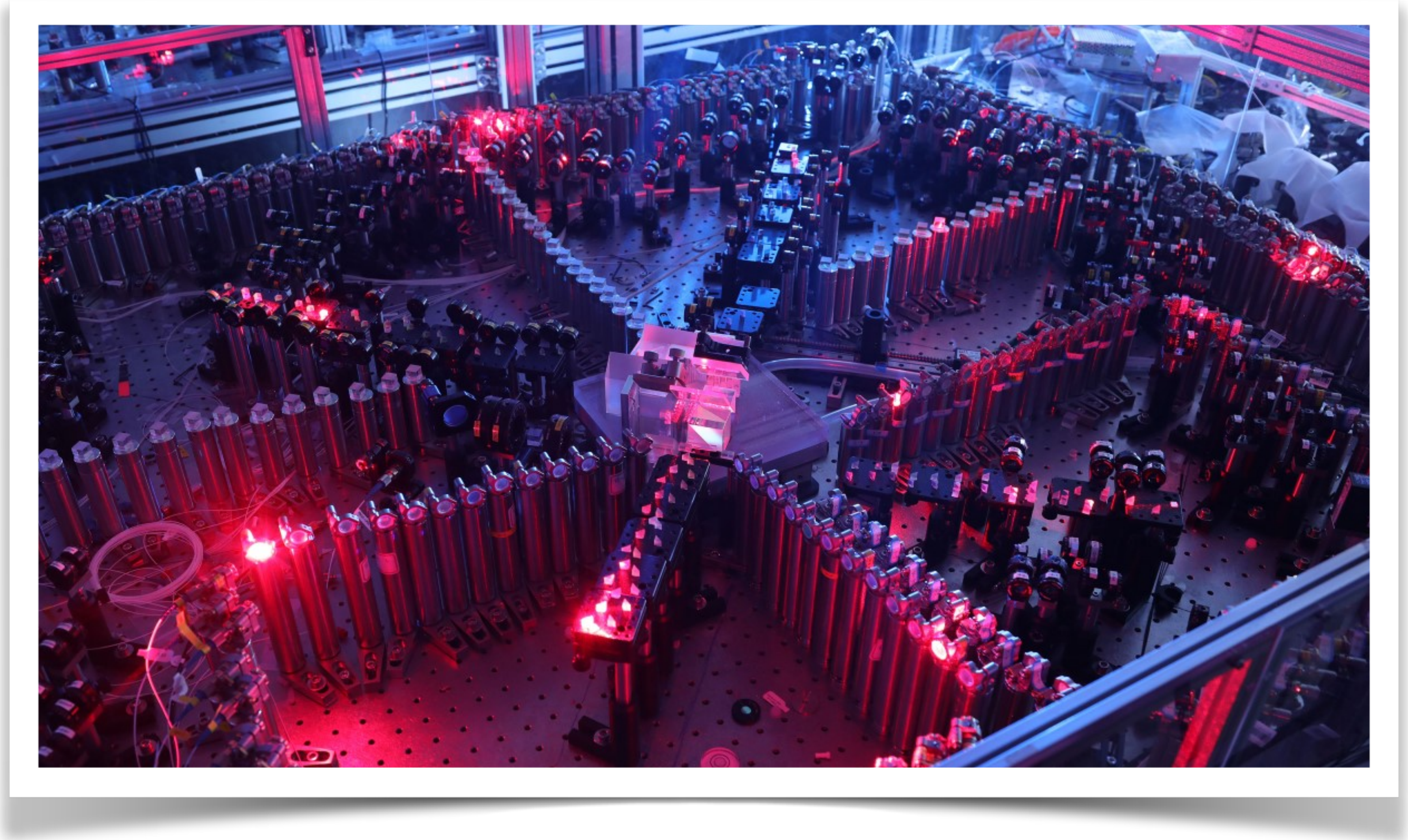
- Landmark quantum computational advantage experiments



[Zhong et al, Science 2020]

Context and motivation

Bosonic systems

- Bosonic statistics lead to remarkable physics
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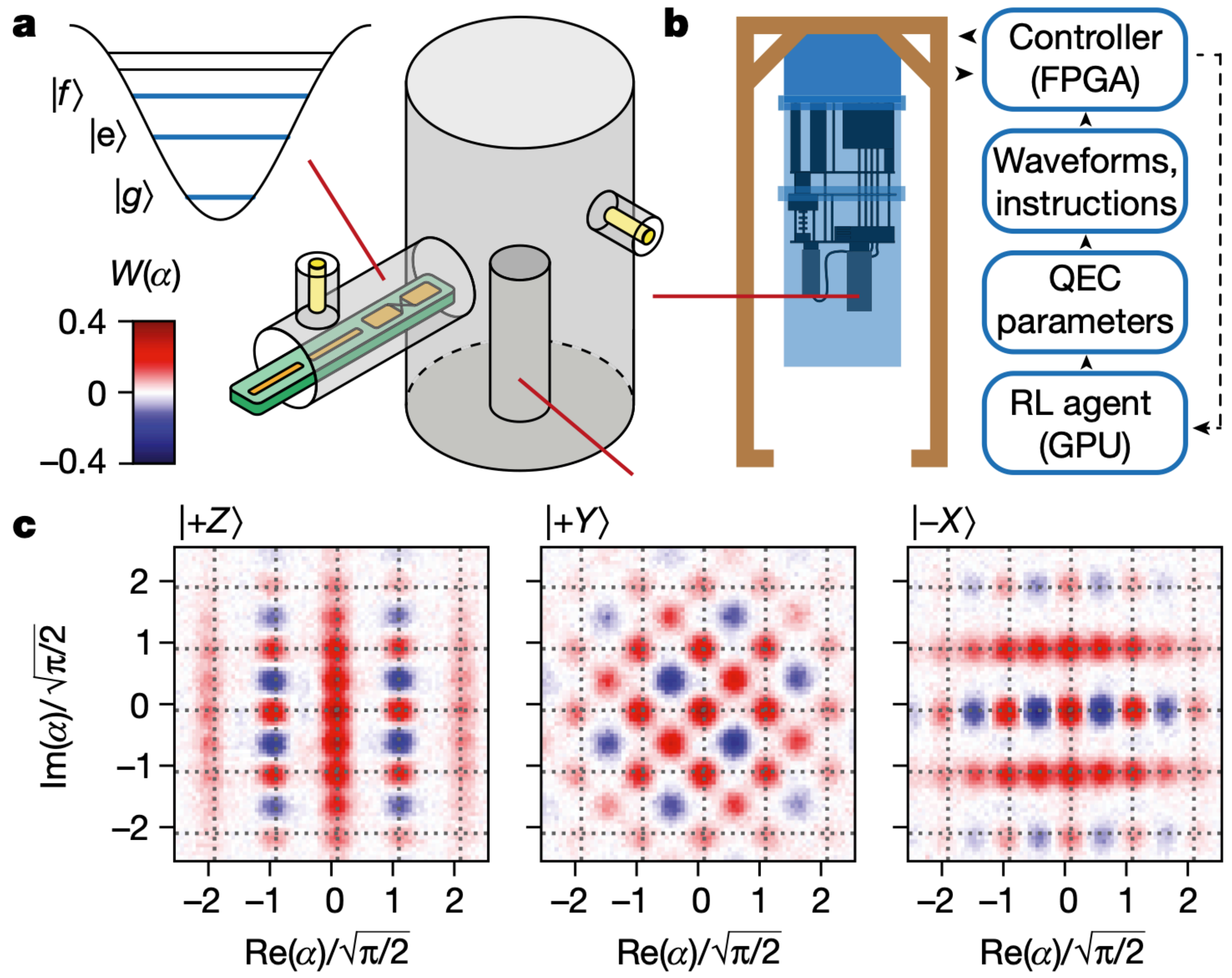
- Crucial for quantum information processing
- Landmark quantum computational advantage experiments
- Candidate platforms for building universal quantum computers



[Sivak et al, Nature 2023]

QUANDELA

ALICE & BOB

aws

PsiQuantum

Nord Quantique

XANADU

...

Context and motivation

Bosonic computations

Quanta magazine

New Quantum Algorithm Factors Numbers With One Qubit

Factoring in polynomial time and *constant space* with standard qubit-boson gates

[Brenner *et al*, arXiv:2412.13164 (2024)]

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The catch: It would require the energy of a few medium-size stars.

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Context and motivation

Bosonic computations

Quantamagazine

What is the role of energy in the computational power of bosons?

The catch: It would require the energy of a few medium-size stars.

Factoring in polynomial time and *constant space* with standard qubit-boson gates

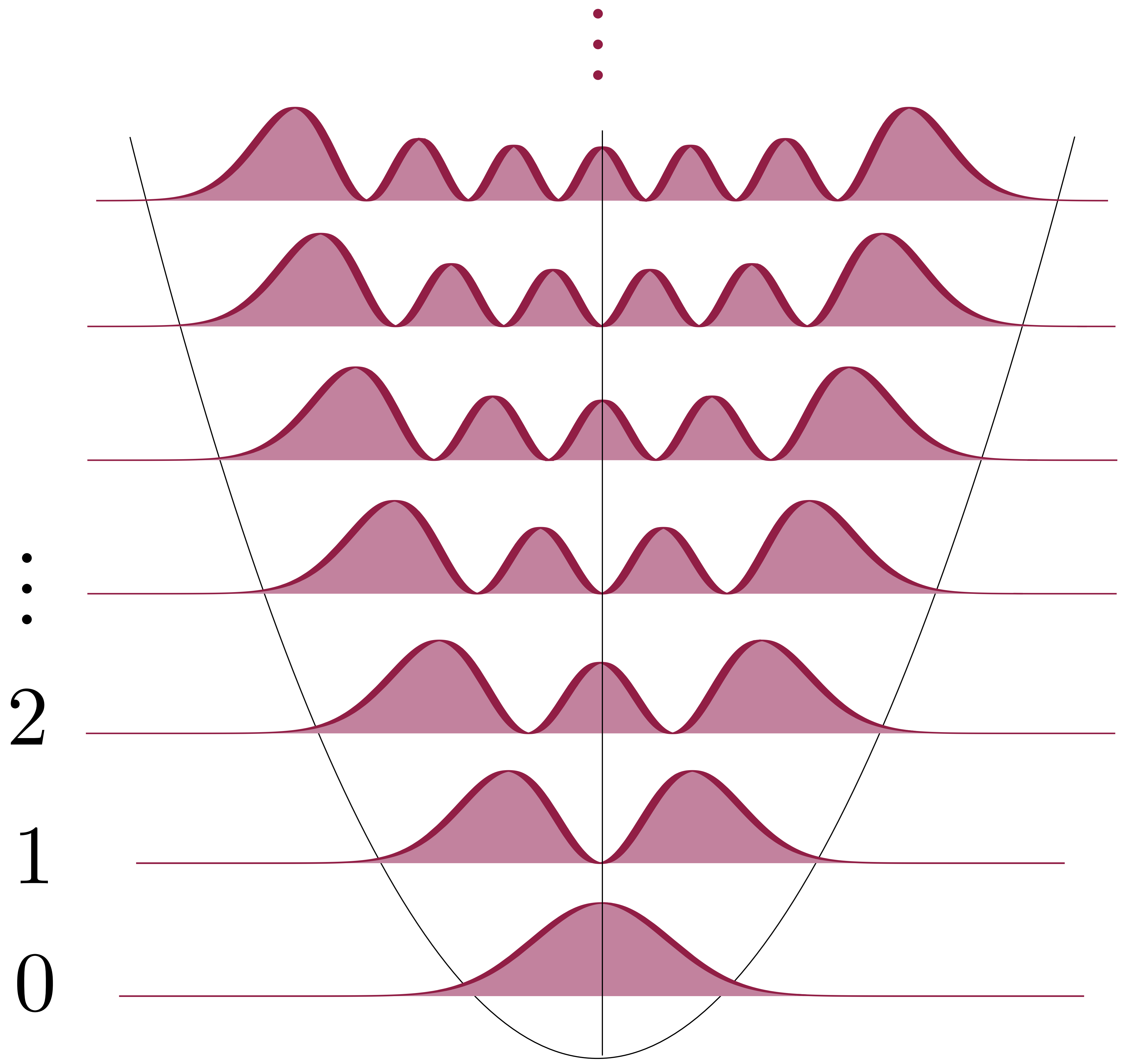
[Brenner *et al*, arXiv:2412.13164 (2024)]

Outline

- Background
- Energy growth in bosonic circuits
- Bounding the computational power of bosons
- The computational power of energy
- Outlook

Background

Bosonic quantum information



Infinite dimension $\{|n\rangle\}_{n\in\mathbb{N}}$

Position and momentum operators $[\hat{q},\hat{p}] = i\hat{I}$

Number operator $\hat{n} = \frac{1}{2}(\hat{q}^2 + \hat{p}^2 - 1)$

Energy: average particle number $N_\rho = \mathrm{Tr}(\hat{n}\rho)$

Background

Bosonic quantum gates

Polynomial Hamiltonians $\hat{H} = \mathrm{poly}(\hat{q}_1, \hat{p}_1, \ldots, \hat{q}_m, \hat{p}_m)$

Standard gate set $\{\underbrace{e^{i\theta(\hat{q}^2+\hat{p}^2)}, e^{ir\hat{q}_1\hat{q}_2}, e^{it\hat{q}}}_{\text{Gaussian}}, \underbrace{e^{i\gamma\hat{q}^3/3}}_{\text{non-Gaussian}}\}$ *Intuition: Clifford and non-Clifford gates*

[Lloyd & Braunstein, PRL 1999]

Background

Bosonic quantum gates

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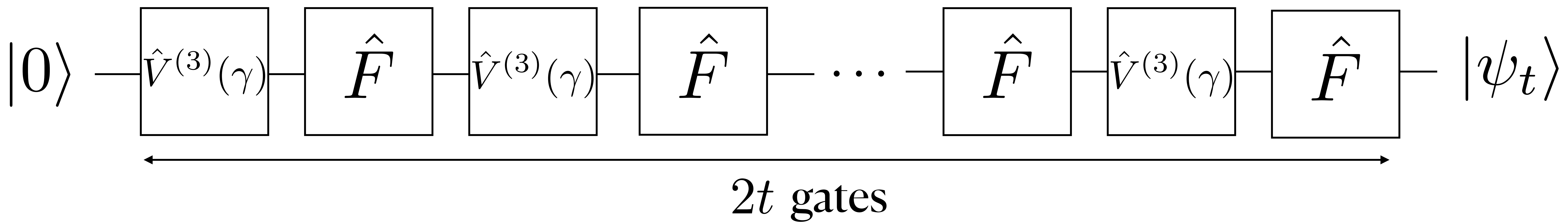
Fourier gate $\hat{F} = e^{i\frac{\pi}{4}(\hat{q}^2+\hat{p}^2)}$ *Intuition: Hadamard gate*

Cubic phase gate $\hat{V}^{(3)}(\gamma) = e^{i\gamma\hat{q}^3/3}$ *Intuition: T gate*

[Lloyd & Braunstein, PRL 1999]

Energy growth in bosonic circuits

Energy growth in bosonic circuits



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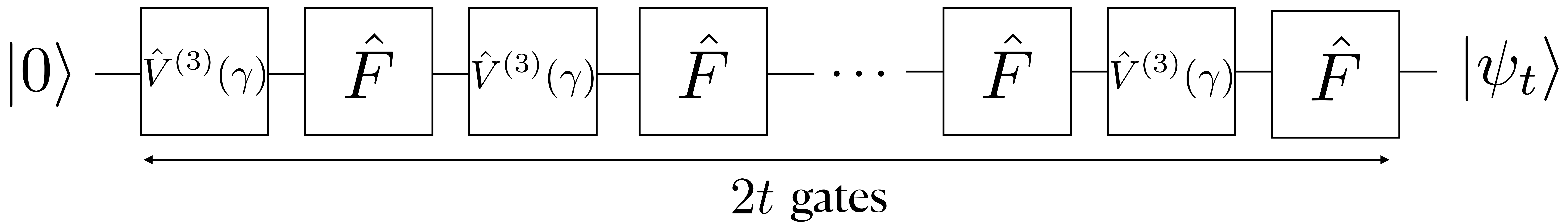
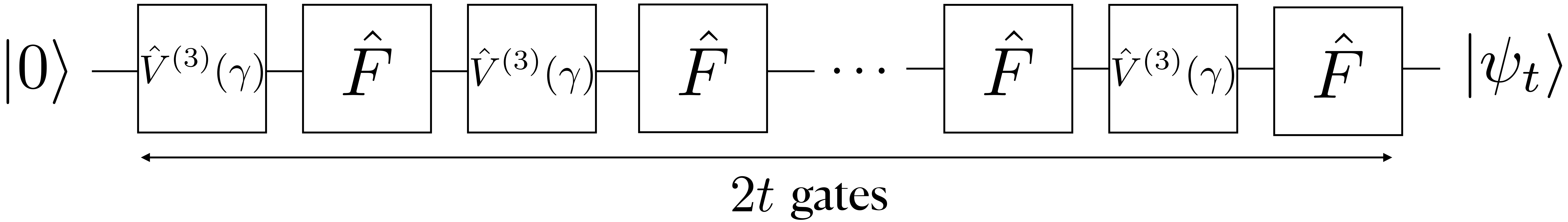


$$\langle\psi_t|\hat{N}|\psi_t\rangle > \left[\frac{1}{12}\left(\frac{\gamma^2}{6}\right)^{2^t-1}\right](2^t)^{2^t}$$

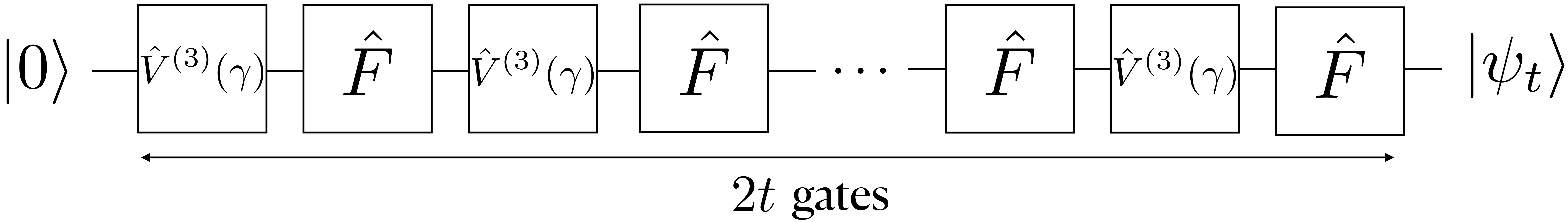
Energy growth in bosonic circuits



$$\langle\psi_t|\hat{N}|\psi_t\rangle > \left[\frac{1}{12}\left(\frac{\gamma^2}{6}\right)^{2^t-1}\right](2^t)^{2^t}$$

Energy grows doubly exponentially fast!

Energy growth in bosonic circuits



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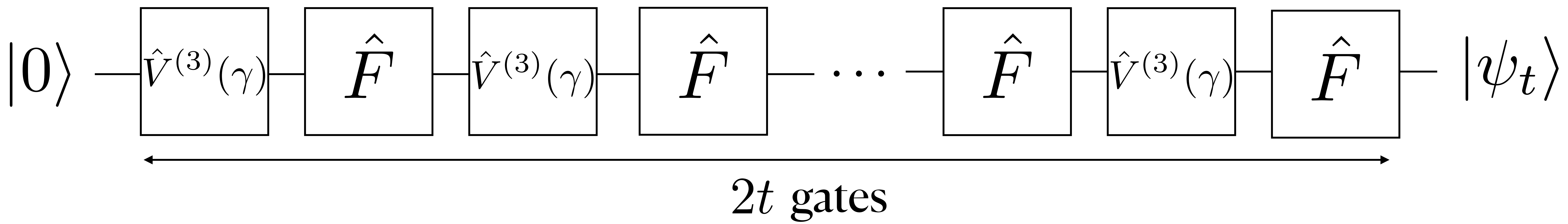
A galactic algorithm…

Energy growth in bosonic circuits



Intuition: repeated squaring

$$\langle\psi_t|\hat{N}|\psi_t\rangle > \left[\frac{1}{12}\left(\frac{\gamma^2}{6}\right)^{2^t-1}\right](2^t)^{2^t}$$

$$\hat{n} = \frac{1}{2}(\hat{q}^2 + \hat{p}^2 - 1)$$

$$\hat{F}^\dagger \hat{q} \hat{F} = -\hat{p}$$
$$\hat{F}^\dagger \hat{p} \hat{F} = \hat{q}$$

$$\hat{V}^{(3)}(\gamma)^\dagger \hat{q} \hat{V}^{(3)}(\gamma) = \hat{q}$$
$$\hat{V}^{(3)}(\gamma)^\dagger \hat{p} \hat{V}^{(3)}(\gamma) = \hat{p} + \gamma \hat{q}^2$$

Energy grows doubly exponentially fast!

Energy growth in bosonic circuits

Other weird facts about energy growth:

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- For some polynomial Hamiltonians, the energy diverges arbitrarily fast

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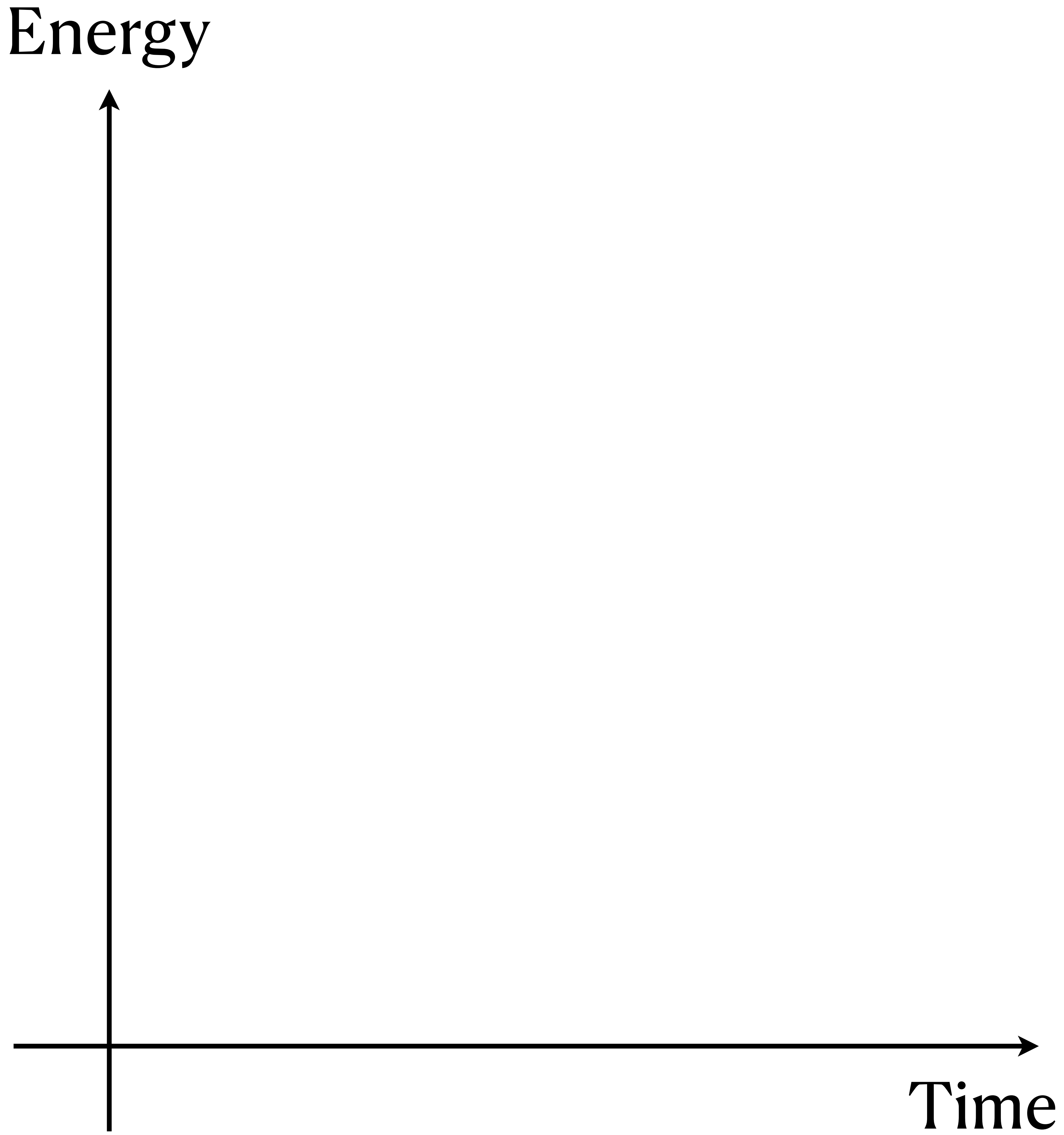
Energy growth in bosonic circuits

Other weird facts about energy growth:

- For some polynomial Hamiltonians, the energy diverges arbitrarily fast
- For some polynomial Hamiltonians, the energy is always infinite
- The problem of deciding whether a computation based on polynomial Hamiltonians will reach infinite energy is undecidable...

Intuition: reduction to the undecidability of Diophantine equations

Bounding the computational power of bosons



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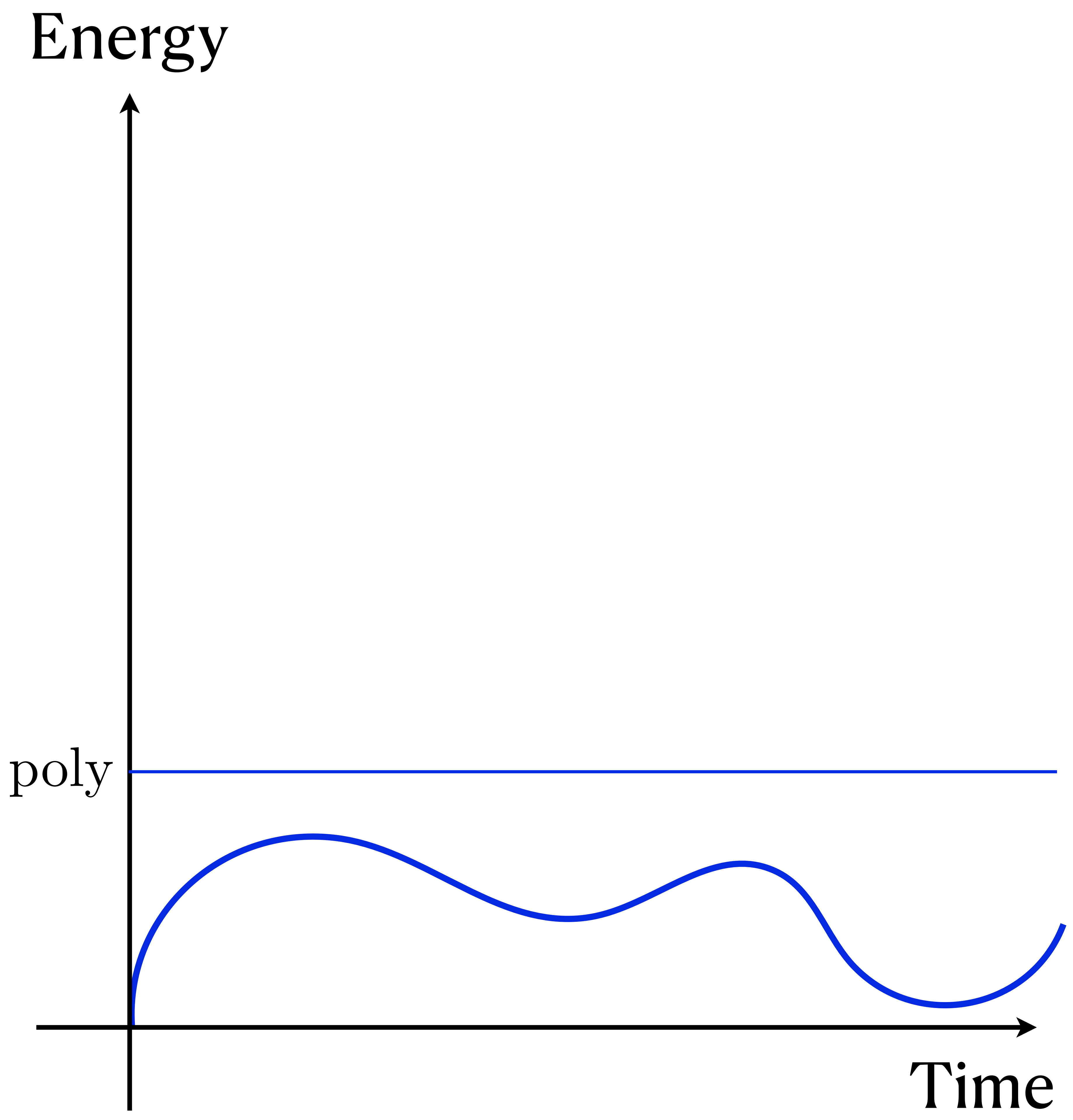
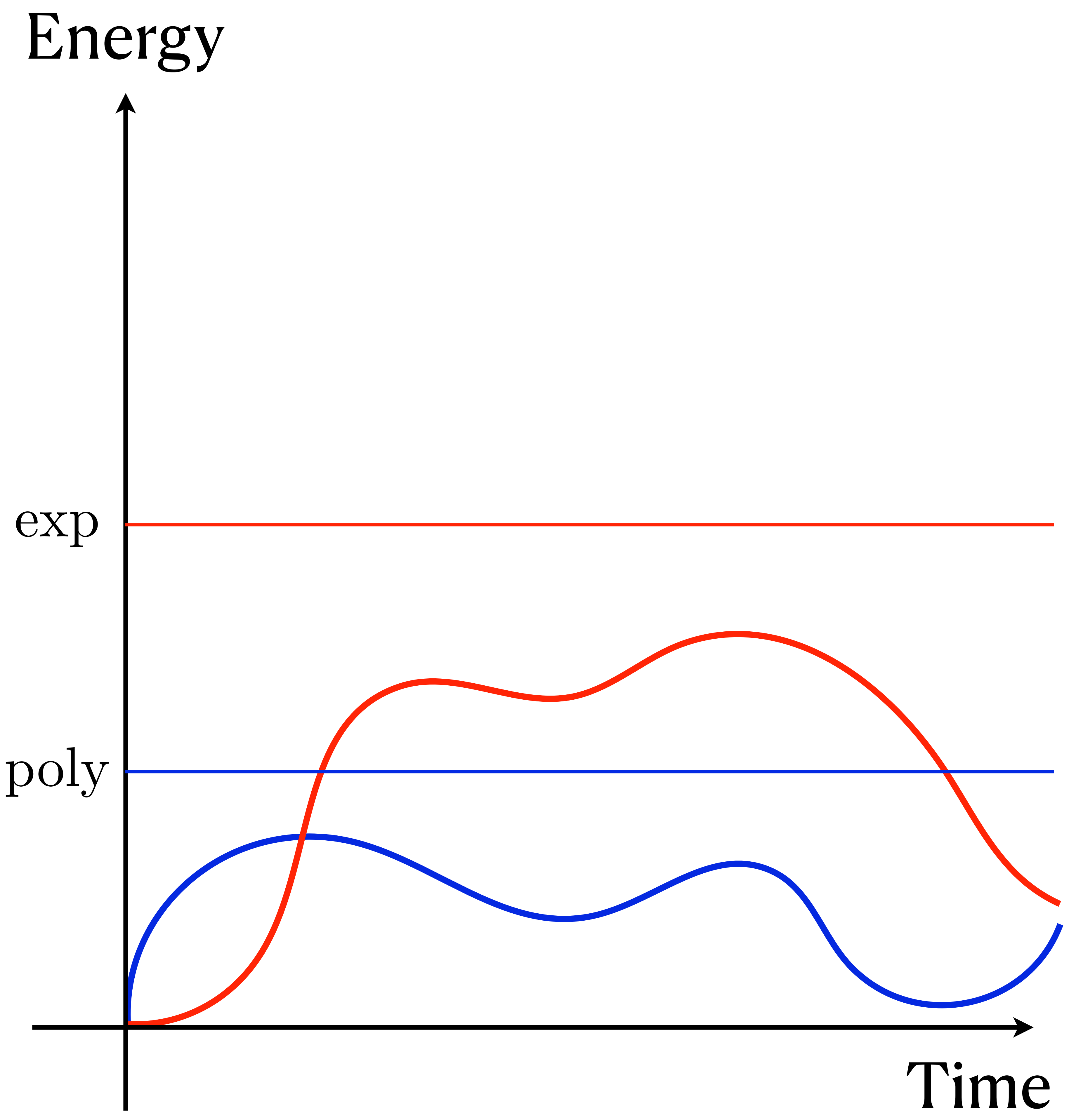


Energy throughout the computation is polynomially bounded $\subset$ BQP

(any polynomial gate set)

Bounding the computational power of bosons



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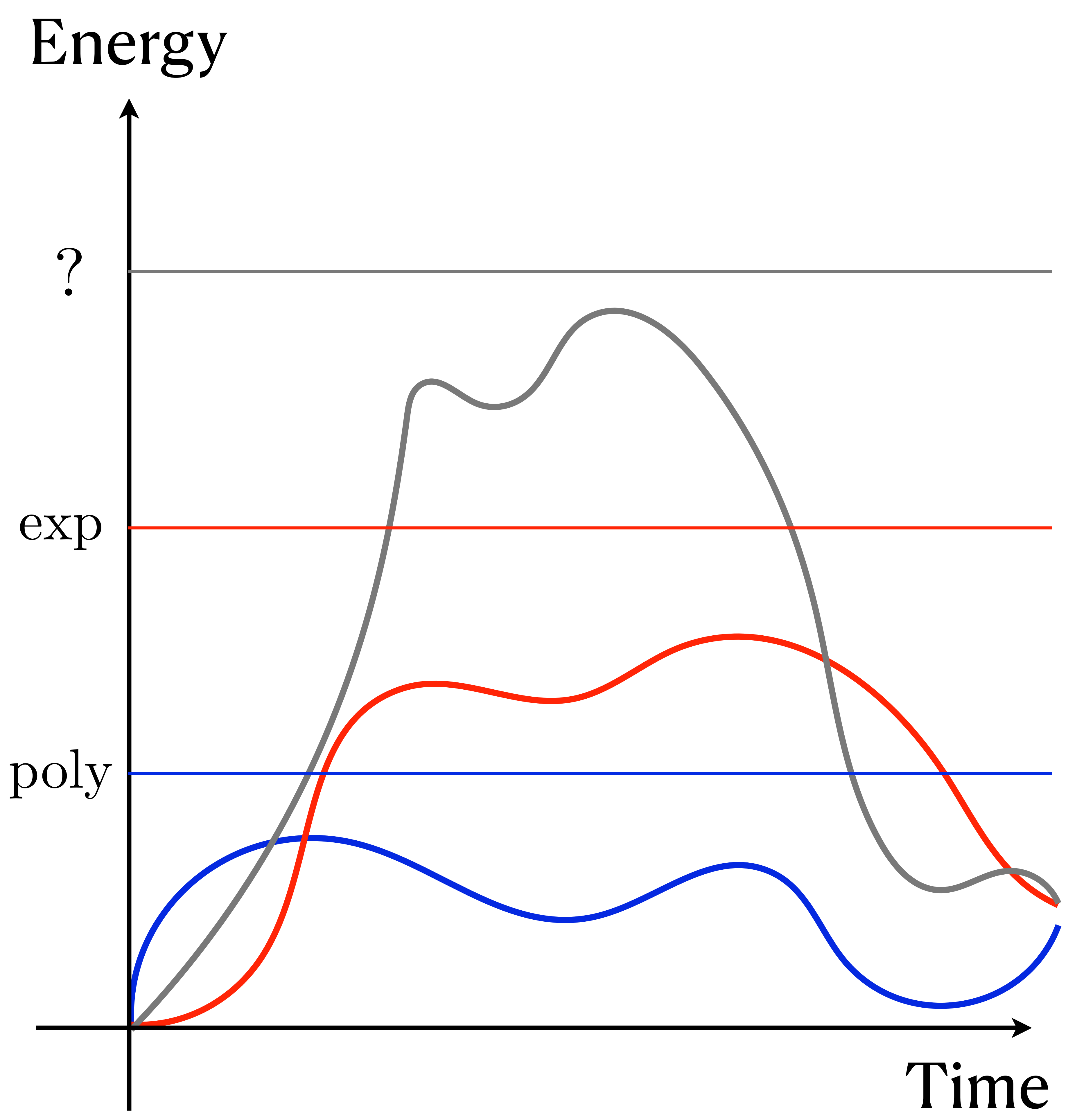
Energy throughout the computation is exponentially bounded (Gaussian + cubic gates) $\subset$ PP

Bounding the computational power of bosons



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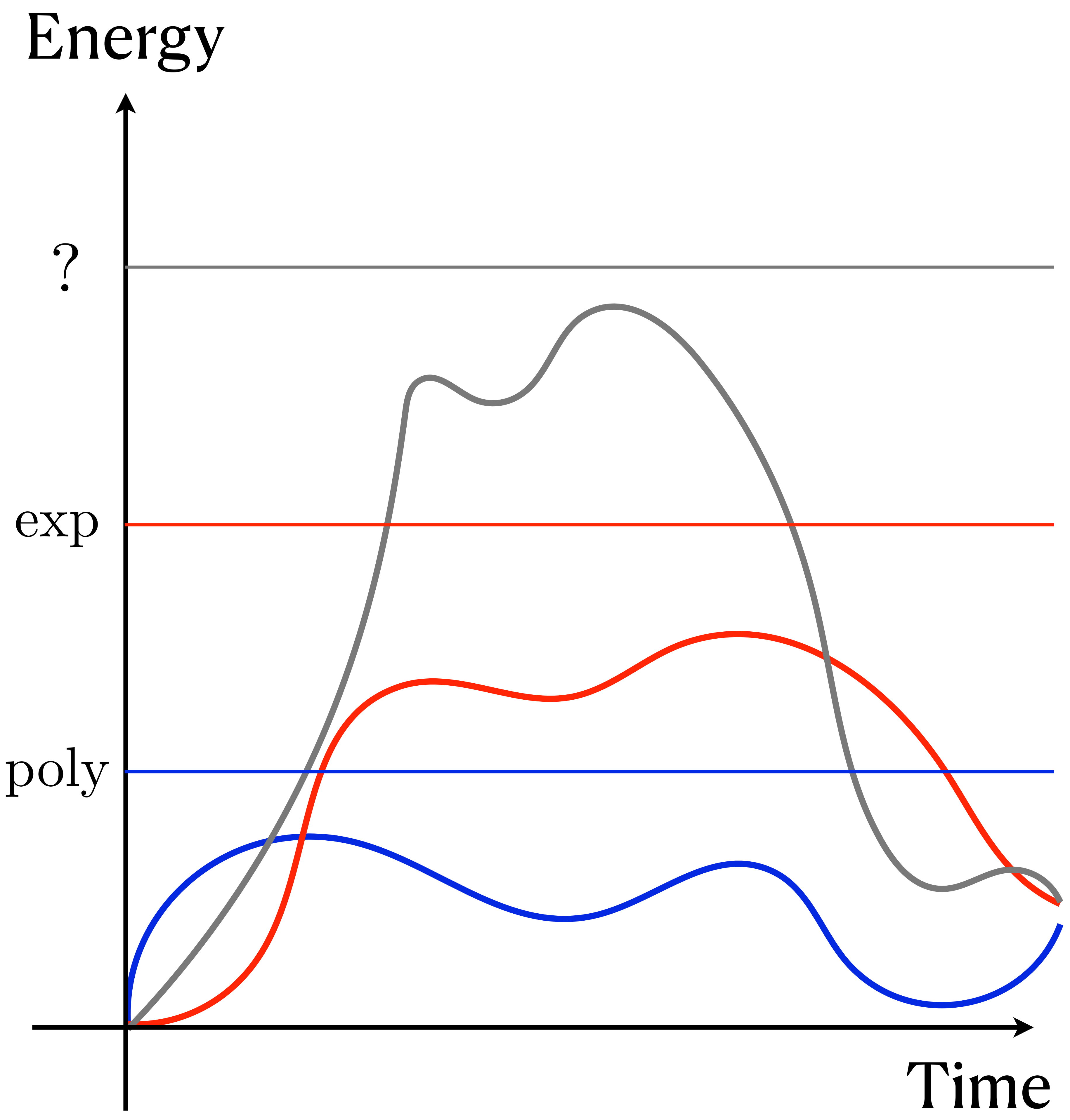
Energy moments throughout the computation are uniformly bounded (any polynomial gate set) $\subset$ R

Bounding the computational power of bosons



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Bounding the computational power of bosons

Energy throughout the computation is exponentially bounded

- Classical simulation of Gaussian + cubic circuits
- Input vacuum
- Particle-number measurements

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$\subset$ EXPSPACE
[arXiv:2410.04274]

Intuition: brute-force simulation without energy bound (cut off in number basis)

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Intuition: coherent state decompositions (cf stabilizer rank)

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[arXiv:2501.13857]

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$\subset$ PP
[arXiv:2510.xxxxx]

Intuition: cubic state injection (cf magic state injection)

The computational power of energy

More energy = more computational power

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$$\exp^{(k)}(n) := e^{e^{\cdots^{e^n}}} \qquad (k \text{ times})$$

- With gates based on polynomial Hamiltonians, we can solve $\mathsf{NTIME}(\exp^{(k)}(n))$ in space $O(k)$ while remaining close to energy $\exp^{(k)}(n)$

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Intuition: algorithm for solving Diophantine equations with bounded solutions

Integer solution to $F(x_1, \ldots, x_k) = 0 \longleftrightarrow$ ground state of $H = F(\hat{n}_1, \ldots, \hat{n}_k)$

Outlook

Take-home messages

Outlook

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[Brenner *et al*, arXiv:2412.13164 (2024)] → trading space for energy

see also [Brenner *et al*, arXiv:2509.18854 (2025)]

Our work → trading time for energy

Outlook

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Like time and space, energy is a computational resource

Outlook

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Dramatic energy growth in standard models of bosonic computation

Outlook

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Dramatic energy growth in standard models of bosonic computation

We need to rethink the current models of computation in infinite dimensions

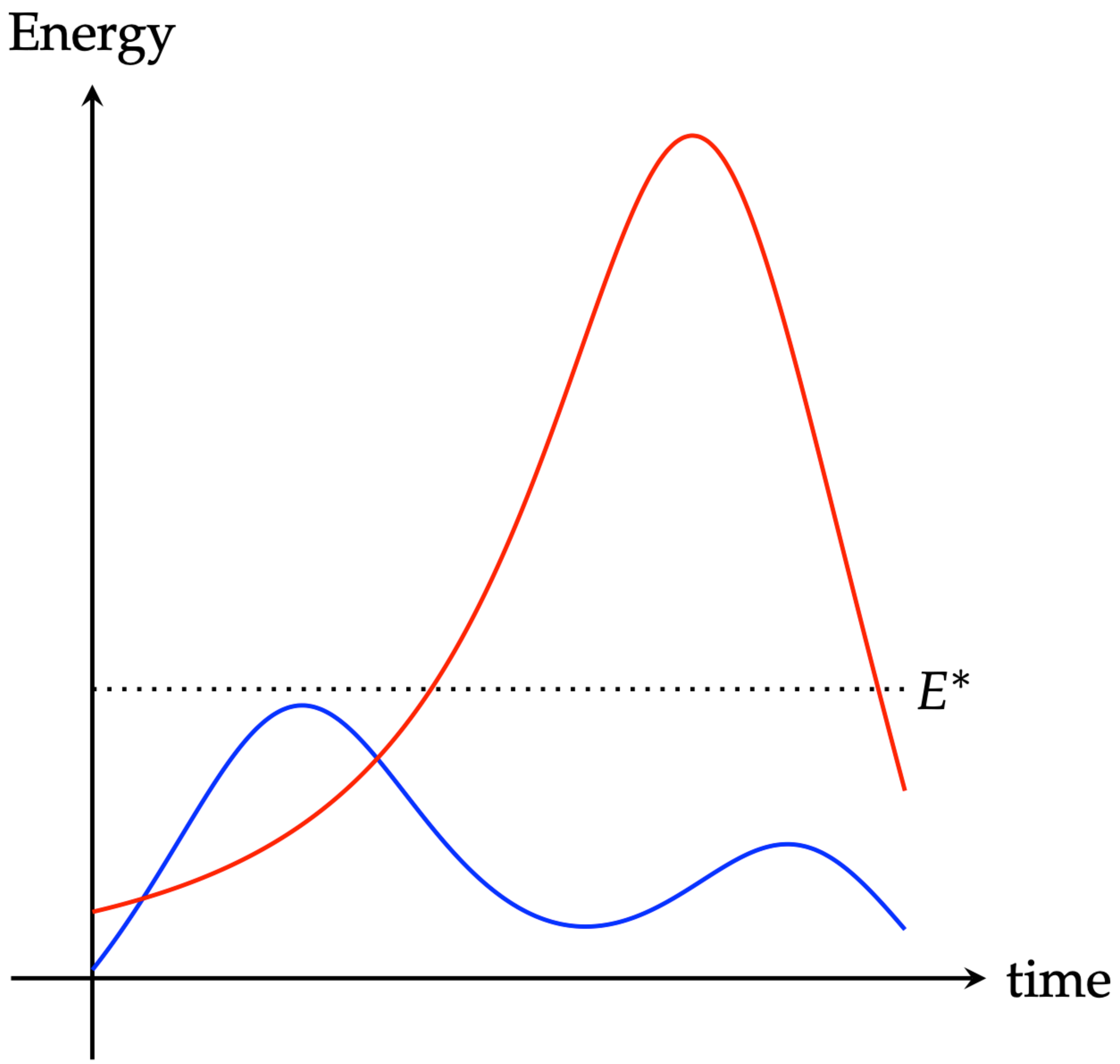
Outlook

Some open questions

What is a good model for infinite-dimensional quantum computations?

Challenging Church–Turing (again): can realistic computations go beyond qubits?

Is computational power fundamentally hardware-dependent?



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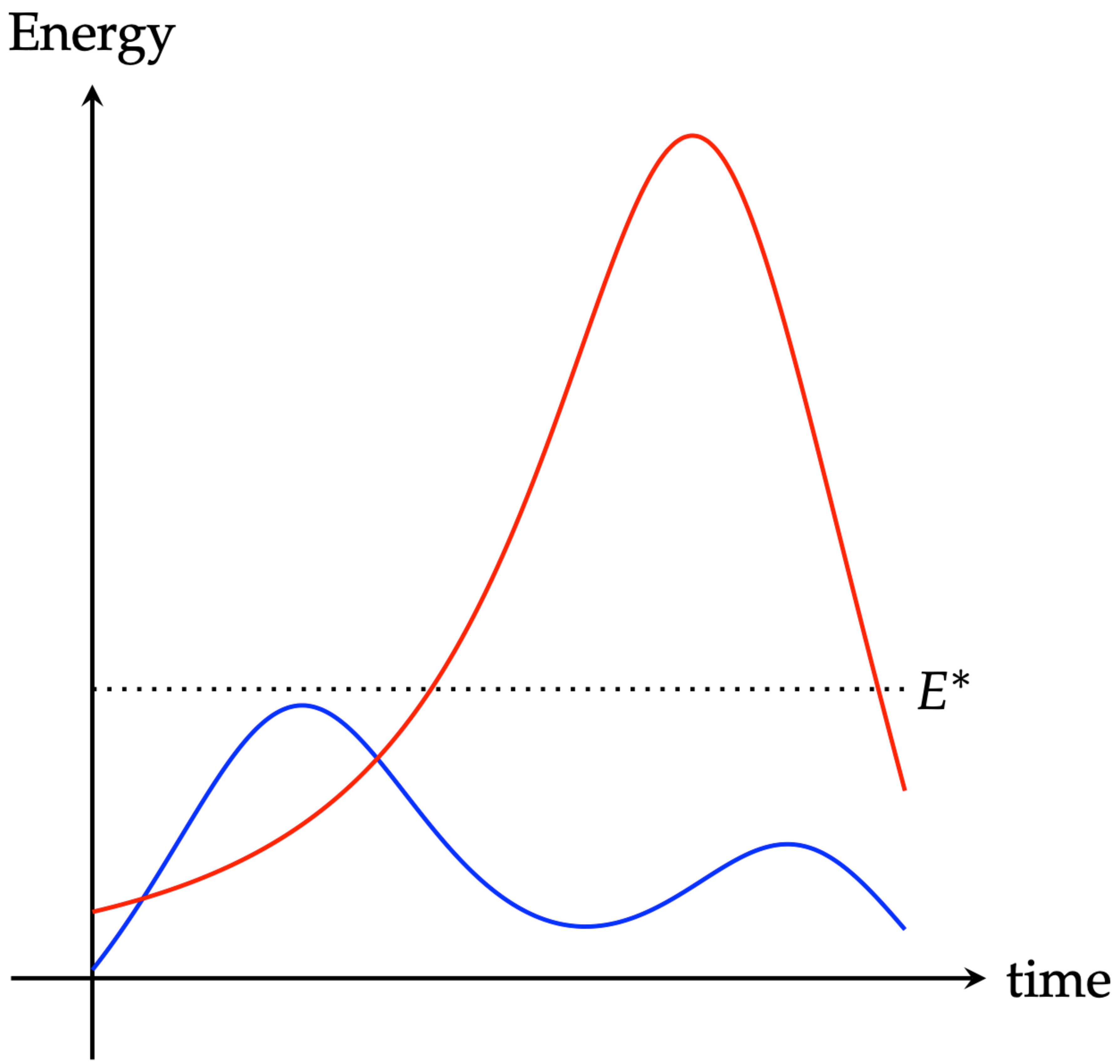

Thank you!