

Biased Random Walk on Spanning Trees of the Ladder Graph

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Abstract Take a two-sided infinite horizontal ladder and pick a random spanning tree with a certain edge weight c for the (vertical) rungs. Now take a random walk on that spanning tree with a bias $\beta > 1$ to the right.

We give an explicit formula for the speed of the biased random walk as a function of both the bias β and the edge weight c . We conclude that the speed is a continuous, unimodal function of β that is positive if and only if $\beta < \beta_c^{(1)}$ for an explicit critical value $\beta_c^{(1)}$ depending on c . In particular, the phase transition at $\beta_c^{(1)}$ is of second order.

We show that another second order phase transition takes place at another critical value $\beta_c^{(2)} < \beta_c^{(1)}$: For $\beta < \beta_c^{(2)}$, the position of the walker fulfills a central limit theorem (after subtracting the linear speed). Finally, we confirm the Einstein relation for the unbiased model ($\beta = 1$) by proving a central limit theorem and computing the variance.