A new hyperbolic sigma model

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Recently, Sabot and Tarrès introduced a new type of vertex reinforced jump process: the $\star$-VRJP. It is defined on a directed graph $G = (\Lambda, E)$ with a special involution $\star : G \mapsto G$, which sends each vertex j to a conjugate vertex $j^\star$, and each edge $\langle ij \rangle$ to a reversed conjugate edge $\langle j^\star i^\star \rangle$. Much like the ordinary VRJP, the $\star$-VRJP is linearly reinforced according to the local time L_t of the walker X_t, but where the ordinary VRJP prefers to jump to where it has been $\mathbb{P}(X_{t+dt} = j \,|X_t = i, L_t) = \beta_{ij} L_t^j$, the $\star$-VRJP prefers to jump to the *conjugate* of where it has been $\mathbb{P}(X_{t+dt} = j \,|X_t = i, L_t) = \beta_{ij} L_t^{j^\star}$.

Also much like the ordinary VRJP, the $\star$-VRJP possesses a variety of remarkable integral identities through its "magic formula" and random Schödinger representation. In the case of the VRJP, through its the deep connection with the $\mathbb{H}^{2|2}$ hyperbolic sigma model, the existence of these integral identities are seen to be a consequence of supersymmetric localisation: this naturally raises the question if there exists a "$\star$-sigma model" counterpart to the $\star$-VRJP to give a similar supersymmetric explanation. In this talk, I will introduce this new hyperbolic sigma model, the $\mathbb{H}_\star^{2n+1|4m}$-model, which is, in a sense, a complexification of the ordinary $\mathbb{H}^{n|2m}$-model, and will present several new isomorphism theorems which connect it to the $\star$-VRJP. Joint work with Sabot and Tarrès.