Comments on mathematics writing

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Last updated: February 2, 2022

By the time you have progressed to the later stages of any mathematics project or thesis, you spend most of your day writing. Despite the importance of this activity, many mathematicians have received little training, and write difficult and ineffective prose. Few recognize how much hard work is required to write well.

Here are some comments from my own point of view. My training came from general humanities and language courses (not specifically writing classes), from following the example set by good communicators (be they mathematicians or not), and from feedback by readers and professional editors. Good general advice - not specific to scientific writing let alone mathematics - can be found in 'The Elements of Style' by Strunk and White [SW]. There are various guides or essays on mathematics writing, but I have found them less useful. If I had to recommend any such text, my first choice would be 'Mathematical Writing' by Donald E. Knuth, Tracy Larrabee, and Paul M. Roberts [KLR]. The advice in paragraph 1 is particularly valuable; and the poor and improved version of the same proof in paragraphs 2 and 3 is a nice introductory example for the impact of good writing.

The main problem of writing about mathematics is the same as when writing about anything:
- you must have something to say,
- you must say it to a particular audience,
- you must organize what you say and in which order,
- you must write it and re-write it several times,
- and you must invest a lot of effort into seemingly 'tedious' things like mathematical notation, using the right words, punctuation, correct attribution of citations and correct form of references, reading flow, and the global structure of your text.

Here is some advice. It does not replace a good humanities background or writing course.

1) **Think carefully about your mathematical notation.** You need to do this before you start typing any first draft. There is a fundamental difference between correct notation and good notation. You cannot just name a mathematical quantity the way you want - even if this makes your definitions and formulae technically speaking correct. Otherwise no-one will understand you. Good notation must be simple, memorizable, in line with conventions, and suited to best bring out the new ideas in your text. A common mistake by inexperienced writers is to stick thoughtlessly to any ad hoc notation they used when first developing an idea by doodling around on paper.

2) **Always introduce key definitions and ideas in words before stating them in formulae.** This helps the reader to understand what is going on. Neat examples illustrating this point can be found in [KLR]. A common mistake by writers unsure of their English is to skip this step, in order to avoid the next mistake listed below (poor wording). But this is even worse than poor wording. It makes your text unreadable.

3) **Write correct English.** Typically, English is not your first language, but if you write a paper, report, or thesis in English (and most mathematicians nowadays do), you must use
- correct words
- correct grammar
- correct punctuation.

Every formula must be part of a sentence. A common mistake is to violate grammar or punctuation rules just because you switch between ordinary language and mathematical formulae in a sentence. A single missing comma does not convert a correct proof into a wrong one, but consistent mistreatment of such things undermines the confidence and patience of the reader, and has a large effect.

Another mistake is to be sloppy in your choice of words. They are just as important as the formulae. Unlike the latter, they need not tell the whole story, but - like the latter - they must be correct. Otherwise wording and formula contradict each other, undermining your results.

The requirement to write correct English applies not just to your final article or thesis, but already to first drafts, or even just working notes which you show to your supervisor or co-workers. While it is fine, and even productive for reasons of efficiency, to leave gaps in such texts (examples: 'To do: investigate this issue', 'To do: explain this notion', 'To do: compare with other approaches'), you need to write correctly.

Your level of English will typically be adaequate to do so. The workflow is the same as for typing mathematical formulae: proofread until correct. If necessary, get a friend to help you with this task.

4) **Try to write good English.** Achieving it may require lifelong hard work, but at least you should try. Not trying impedes the dissemination of all the mathematics you worked so hard on during your project, which is a pity.

Stick with basic rules like the ones in [SW]. The three most important ones are:
- use the active voice
- omit needless words
- follow the principle of parallel construction.

Excellent examples which illustrate the power of these rules can be found in [SW]. I have found that mathematicians are particularly prone to violate the first and the last rule, in a misplaced attempt to hide their personality or to vary their style. The second rule, by contrast, is familiar to mathematicians in the context of stating mathematical theorems. You should also follow it elsewhere. In the words of Strunk and White [SW]: "A sentence should contain no unnecessary words ... for the same reason that a drawing should have no unnecessary lines and a machine no unnecessary parts. This requires not that the writer make all his sentences short, or that he avoid all detail ..., but that every word tell." The last rule says that you should express similar ideas in similar form. To see why, look at the following basic example from [SW].

Formerly, science was taught by the textbook method, while now the laboratory method is employed.	Formerly, science was taught by the textbook method; now it is taught by the laboratory method.

The left version makes it more difficult for the reader to quickly grasp the content, because of the unmotivated change of grammatical subject (science versus the laboratory method) in the middle of the sentence. Such writing is particularly counterproductive in mathematical texts, where the content itself can already be challenging.

As background education, regularly take time out to read some well written mathematical text, with attention to why it is well written. An example you can start with is the PhD thesis by Richard Feynman [F]. The focus on important points, reading flow, and seamless integration of mathematical formulae into the text can be appreciated without any knowledge of the subject matter (quantum mechanics) .

Two basic iteration loops to improve your own text are:
- Read with the eyes, improve and concisify, repeat until convergence.
- Read out loud, improve the reading flow, repeat until convergence.

5) **Pay attention to the global structure of your text.** Ideally you should have learned such things at school.

Begin with an Introduction which
- (sparsely) introduces the topic
- announces the problem formulation and your motivation for studying it
- describes your overall approach
- lays out the plan of the paper/thesis/document.

In the main document, think carefully about how to split the work into coherent sections. At the beginning of each section, announce what you want to achieve here. Make sure that each section has a concise line of mathematical argumentation. For example, think carefully about splitting up a longer proof into lemmas, the proof of lemmas into steps, etc. If the overall flow of logic is complicated, outline it first.

Summarize your overall findings concisely and in plain language at the end of the document. This is the last impression you make on your reader. A common mistake by mathematicians is to skip the conclusions. This is a variant of the mistake described under 2). It is your job, not that of the reader, to distill overall conclusions from the collection of detailed results/theorems/ simulations which you presented in the main text.

Further reading

[SW] William Strunk Jr. and E. B. White, *The Elements of Style*, 4th edition, Longman, 2000

[DLR] Donald E. Knuth, Tracy Larrabee, and Paul M. Roberts, *Mathematical Writing,* Mathematical Association of America, 1996

[F] Feynman's Thesis - *A New Approach To Quantum Theory*, edited by Laurie M. Brown, World Scientific, 2000

Acknowledgements

I am indebted to Anne Dumenil, Benedikt Graswald and Arseniy Tsipenyuk for helpful comments on a first draft of this text.