In exercise sheet 1 we *almost* proved the following essential tool.

**Theorem 0.1** (Contraction principle)**.** *Assume that the random variables $X^n$ satisfy an LDP in a topological space $\mathcal{X}$ with good rate functional $I : \mathcal{X} \to [0,\infty]$. Define $Y^n := \phi(X^n)$ where $\phi : \mathcal{X} \to \mathcal{Y}$ is a continuous mapping between the two topological spaces $\mathcal{X}$ and $\mathcal{Y}$. Then $Y^n$ satisfies an LDP with good rate functional $J : \mathcal{Y} \to [0,\infty]$,*

$$J(y) := \inf_{x\in\mathcal{X}:\phi(x)=y} I(x).$$

*Note*: This does not work without the goodness, since $\phi$ maps compact sets to compact sets, but only the pre-image of a closed set is closed!

1. Let $X_1, X_2, \ldots$ be iid random variables in $[-1,1]$ with $\mathrm{Law}(X_1) = \nu$, and let $S_n := \sum_{i=1}^n X_i$ be the empirical sum. Use Sanov's Theorem to prove that $\frac{1}{n}S_n$ satisfies an LDP in $[-1,1]$, with an expression of the rate functional that differs from the one from Cramér's Theorem. (There is no need to prove that the two rate functionals are the same; this follows from uniqueness of large deviation rate functionals!)